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Contact Mechanics in OOFEM

OOFEM Meeting Presentation

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Contents

Motivation: Contact Mechanics

- Initially studied: Heinrich Hertz, 1881, contact of elliptic elastic rigid bodies without friction
- Only selected special cases have analytical solutions
- Many practical applications (mechanical, civil engineering, material science)
- Since the 1960s: FEM and contact algorithms
- Progress in hardware enables solution of more complicated contact cases
- Many cases and many approaches to FEM simulation
  - Contact with or without friction
  - Different FEM discretizations
  - Different handling of the contact condition

- We have a system in equilibrium, see figure
- Equilibrium expressed in terms of energy:

$$W(u) = \frac{1}{2}ku^2 - mgu \to \min \qquad (1)$$

- Introduction of an additional contact condition $\to$ deformation constraint - penetration function:

$$c(u) = u - h \leq 0 \qquad (2)$$

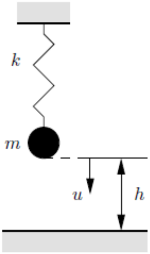


Figure: A mass on a spring with a contact condition

- Introduction of the contact condition into the energy functional: two approaches
  - Lagrangian multiplier (LM) method:

$$L(u, \lambda) = W(u) + \lambda c(u) \tag{3}$$

  - Penalty method:

$$W_p(u) = W(u) + \frac{1}{2} p c^2(u) \tag{4}$$

- Lagrangian multiplier method
  - Allows for a precise solution
  - A new variable introduced for each contact point
  - The very existence of this variable is contact-condition-dependent
- Penalty method
  - Imprecise solution (precise for $p \to \infty$)
  - Large penalty precise enough, yet unwieldy for the solver
  - Precision vs ease of solving conflict

- FEM: physical space discretized into elements and nodes
- Contact condition introduction depends on what contacts with what
- NTN - node to node
  - easiest, simple projection
  - linear geometry only
- NTS - node to segment
  - nonlinear geometries possible
  - more complicated contact search
  - segment, typically, is an element boundary
  - analytical function as a segment - simulates a rigid obstacle
- STS - segment to segment - future

Implementation overview

- node-2-node contact conditions functional in 2D and 3D (penalty and LM)
- node-2-segment contact conditions for linear 2D (penalty and LM)
- available contact segments include element edges and analytical functions (circle, polynomial)
- node-2-segment contact conditions for geometrically nonlinear 2D (plane strain) and 3D simulations - only penalty method for now

Node-to-Node Contact

Overview

- Implementationally simple
  - two new classes: *Node2NodePenaltyContact* and *Node2NodeLagrangianMultiplierContact*
  - inherited from *ActiveBoundaryCondition*
- Node pairings are user-specified
- Unsuitable for geometrical nonlinearity for obvious reasons

Node-to-Node Contact

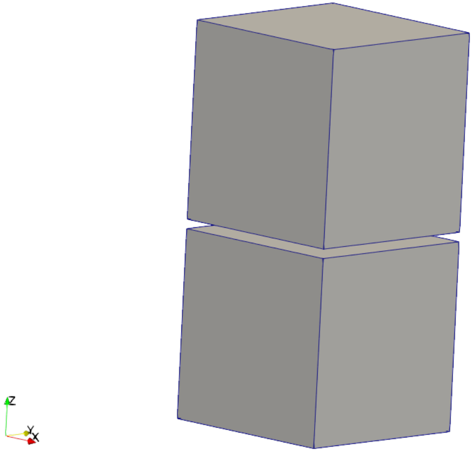
Input File Example


```
# BCS
BoundaryCondition 1 loadTimeFunction 1 values 3 0.0 0.0 0.0 dofs 3 1 2 3 set 1
BoundaryCondition 2 loadTimeFunction 1 values 3 0.0 0.0 -0.05 dofs 3 1 2 3 set 2
n2npenaltycontact 3 loadTimeFunction 2 penalty 1.e8 masterset 2 slaveset 3 usetangent normal 3 0 0 1
# TIME FUNCTIONS
PiecewiseLinFunction 1 npoints 3 t 3 -1 0. 500 f(t) 3 0 1 501
ConstantFunction 2 f(t) 1.0
# SETS
# fixed nodes
Set 1 nodes 8 1 2 3 4 15 16 17 18
# nodes to be moved = masterset
Set 2 nodes 4 11 12 13 14
# slaveset
Set 3 nodes 4 5 6 7 8
```

Figure: Input file for node-to-node contact

The optional *normal* keyword defines a prescribed normal direction (master to slave) of the contact, overwrites the usual procedure for computing it from reference node coordinates

Node-to-Node Contact
The Resulting Analysis



Node-to-Segment Contact
Class Structure

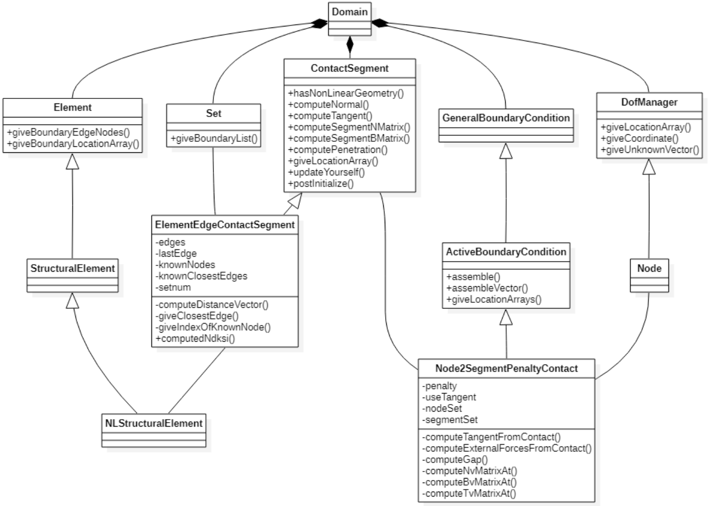


Figure: Class structure in OOFEM for node-to-segment contact

Node-to-Segment Contact

Equations

- The universal equations for the internal forces and tangent stiffness in node-to-segment contact:

$$\boldsymbol{f}_c = \int_{\Gamma_c} p g_c \boldsymbol{N}_v \,\mathrm{d}\Gamma \tag{5}$$

$$\begin{aligned} \boldsymbol{K}_c &= \int_{\Gamma_c} p \boldsymbol{N}_v^T \boldsymbol{N}_v + p g_c \left(\boldsymbol{B}_{v,\alpha} \boldsymbol{D}_{v,\alpha}^T + \boldsymbol{D}_{v,\alpha} \boldsymbol{B}_{v,\alpha}^T \right. \\ &+ \left. \kappa_{\alpha\beta} \boldsymbol{D}_{v,\beta} \boldsymbol{D}_{v,\alpha}^T + g_c m^{\alpha\beta} \bar{\boldsymbol{B}}_{v,\alpha} \bar{\boldsymbol{B}}_{v,\beta}^T \right) \,\mathrm{d}\Gamma \end{aligned} \tag{6}$$

- Division of responsibilities between the contact segment classes, which supply the different submatrices, and the contact condition class which puts it all together

Node-to-Segment Contact

The Contact Condition Classes

- defined in input files similarly to the node-to-node case
- remembers node and segments. In this case, all nodes are tested for contact against all segments
- does not use sets to define nodes and segments

```
# BCS
BoundaryCondition 1 loadTimeFunction 1 values 3 0.0 0.0 0.0 dofs 3 1 2 3 set 1
BoundaryCondition 2 loadTimeFunction 1 values 3 0.0 0.0 -0.005 dofs 3 1 2 3 set 2
n2spenaltycontact 3 loadTimeFunction 2 penalty 1.e4 nodeset 1 9 segmentset 1 1 usetangent
# TIME FUNCTIONS
PiecewiseLinFunction 1 npoints 3 t 3 -1 0. 500 f(t) 3 0 1 501
ConstantFunction 2 f(t) 1.0
# SETS
# fixed nodes
Set 1 nodes 7 1 2 3 4 15 16 17
# nodes to be moved = masterset
Set 2 nodes 1 11
# set of boundaries
Set 3 elementboundaries 2 1 1
```

Figure: Input file excerpt for node-to-segment contact conditions

Node-to-Segment Contact

The Contact Segment Classes

- a new type of object, between elements and materials
- various types
- the boundary segments use boundary sets to enumerate the element boundaries they contain (one segment may contain multiple, typically an entire face of the meshed object)

```
ndofman 12 nelem 2 ncrosssect 1 nmat 1 nbc 3 nic 0 nltf 2 nset 3 ncontactseg 1
# NODES
# Element 1 (lower)
Node 1 coords 3 0 0 0
Node 2 coords 3 1 0 0
Node 3 coords 3 1 1 0
Node 4 coords 3 0 1 0
Node 5 coords 3 0 0 1
Node 6 coords 3 1 0 1
Node 7 coords 3 1 1 1
Node 8 coords 3 0 1 1
# Element 2 (upper)
Node 11 coords 3 0.75 0.75 1.1
Node 15 coords 3 0 0 2.1
Node 16 coords 3 1 0 2.1
Node 17 coords 3 0.5 1 2.1
# ELEMENTS
LSpace 1 nodes 8 8 7 6 5 4 3 2 1 mat 1 crossSect 1 nlgeo 1
LTRSpace 2 nodes 4 17 16 15 11 mat 1 crossSect 1 nlgeo 1
# CONTACT SEGMENTS
linear3delementsurfacecontactsegment 1 boundaryset 3
```

Figure: Input file excerpt for node-to-segment contact segments

Node-to-Segment Contact

The Contact Segment Classes

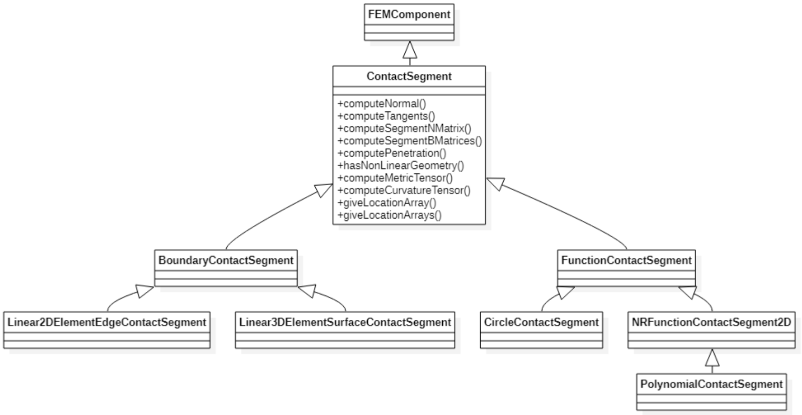


Figure: An UML diagram of existing contact segment classes

Node-to-Segment Contact

Geometrical Nonlinearity

- introduction of geometrical nonlinearity brings some challenges, and necessitates changes and additions to element interpolation classes:
- Closest point projection procedure: implementation of global-to-boundary-local coordinate conversion for 2D and 3D elements - now solved by a universal NR iteration in the contact segment class
- Determination of surface normal in deformed configuration: as a vector cross product of tangential vectors, which have to be provided by element surface - there is now inconsistency in the normal vector direction among different elements and element surfaces (the direction is dependent on the order of nodes in the element definition)

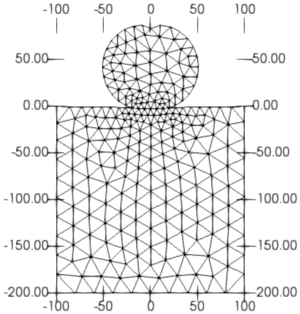
- By Heinrich Hertz, 1881 - formulated the analytical solution
- Conditions:
    - Two elastic bodies are touching by opposite convex surfaces
    - Contact area is very small in comparison to the size of the bodies
    - No friction
- Here a cyllinder and a prism, 2D simulation
- Maximum pressure on the contact area and the contact area width are given analytically as

$$p_0 = \sqrt{\frac{FE}{2\pi R}} \quad (7)$$

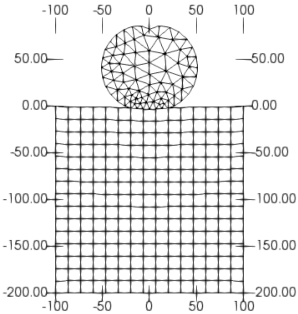
$$a = \sqrt{\frac{8FR}{\pi E}} \quad (8)$$

Tabulka: The Hertz Experiment: Correlation of numerical results for elastic bodies

Computation	Max. Contact Pressure p_0	Contact Area Width a
Analytic	11 337 MPa	19.64 mm
NTN Analysis	11 142 MPa	19.59 mm
NTS Analysis	10 647 MPa	19.55 mm



(a) NTN Discretization

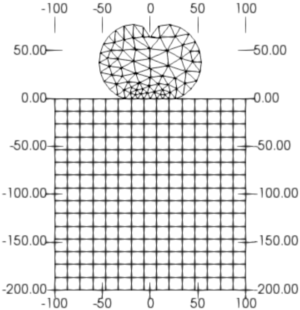


(b) NTS Discretization

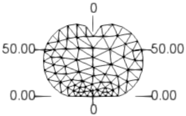
Figure: The Hertz experiment: Comparison of the NTN and NTS discretizations

Tabulka: The Hertz Experiment: Correlation of numerical results for a rigid obstacle

Computation	Max. Contact Pressure p_0	Contact Area Width a
Analytic	20 994 MPa	27.22 mm
Rigid Body	20 556 MPa	26.54 mm
Analytical Function	20 556 MPa	26.54 mm



(a) Rigid body



(b) Analytical function

Figure: The Hertz experiment: Different variants of rigid obstacle simulation

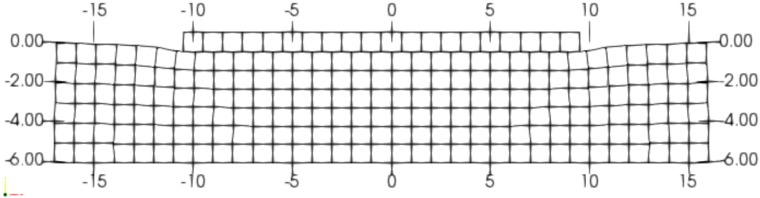


Figure: Deformed mesh 20-198F

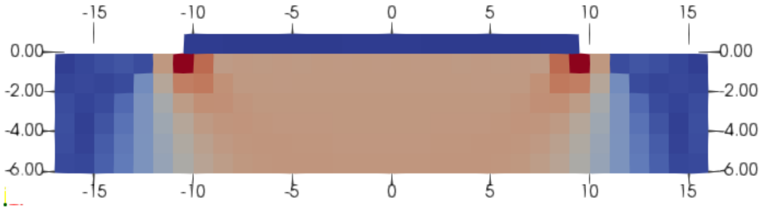


Figure: Stress tensor norm on the 20-198F mesh

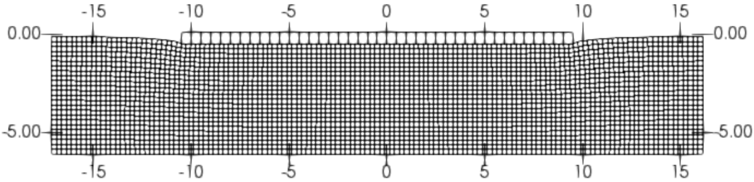


Figure: Deformed mesh 40-3192FX

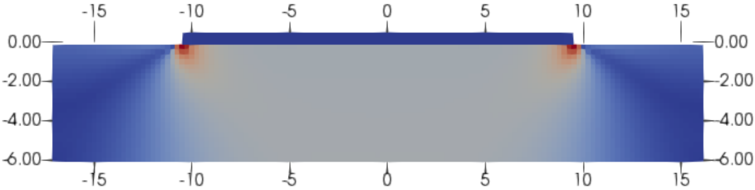


Figure: Stress tensor norm on the 40-3192FX mesh

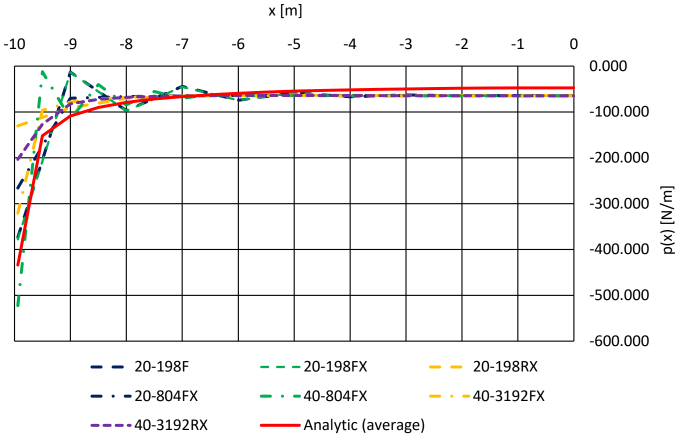


Figure: Rigid flat punch problem: Pressure distribution comparison among all meshes

- Two 2D beams
- Doubled contact condition (each for one set of nodes and the opposite set of segments)

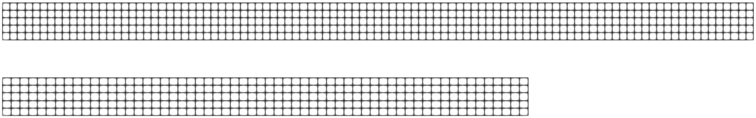
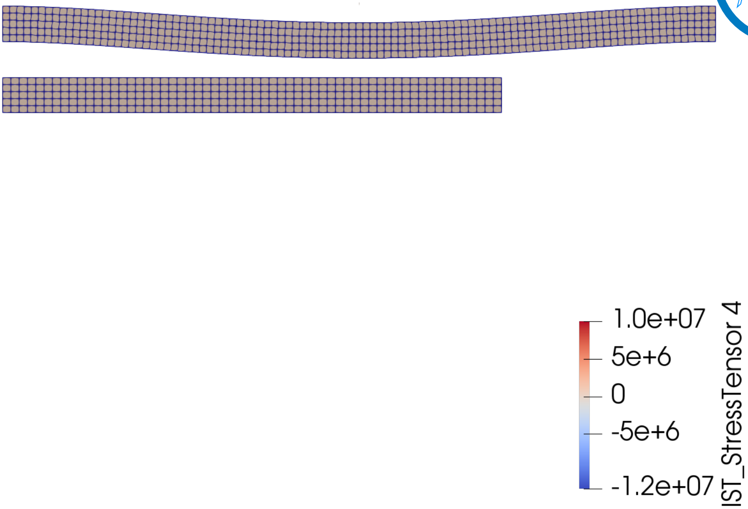


Figure: Mesh

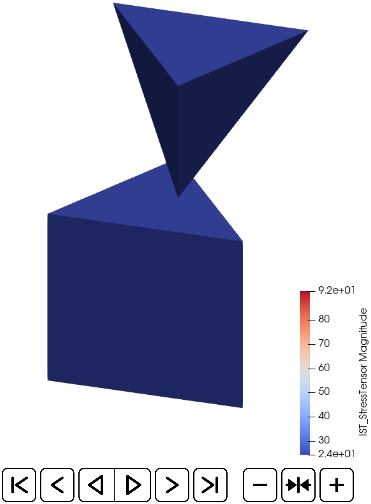
Examples
Geometrically nonlinear 2D contact

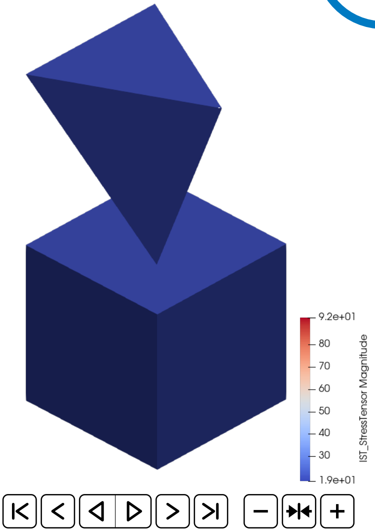


To demonstrate the ability of enforcing contact on the 3D element surface, consider a linear-wedge and linear-brick element as pictured. The triangular surface of the wedge is linear, while the quadrangular surface of the brick is bilinear.

Examples

Geometrically nonlinear 3D contact





Conclusions and Future

- All classes should have in-code documentation now; there are some tests for the simpler cases as well (not for the newest 3D cases)
- The new segment class for 3D and the associated changes in the contact condition could use some code cleanup and optimization

- Lagrangian multipliers have been neglected
- Most urgent extension: either a segment-to-segment condition or a friction model

Thank You for Your attention