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Stochastická nelineární analýza betonových konstrukcí: spolehlivost, vliv velikosti, inverzní analýza

Drahomír Novák

Institute of Structural Mechanics, Faculty of Civil Engineering, Brno University of Technology, Brno Czech Republic

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Outline

- Stochastic techniques for uncertainties simulation
- Software FReET – Feasible Reliability Engineering Tool

Development stimulations:

- Long-term focus of Brno reliability group (Vořechovský, Rusina, Lehký …
- SARA project (Bergmeister, Pukl, Červenka, Strauss …)
  - To combine efficient methods of reliability and nonlinear analysis
  - Software ATENA+FReET=SARA
  - To provide an advanced tool for assessment of real behavior of concrete structures
- Selected types of applications (stochastic nonlinear analysis)

Stochastic techniques for uncertainties simulation

- Introduction – computational demands
- Small-sample simulation of Monte Carlo type
- Imposing statistical correlation
- Simulation of random fields
- Sensitivity analysis
- Reliability analysis
- Inverse analysis

Two main categories of stochastic tasks/approaches

- Approaches focused on the calculation of statistical moments of response quantities (means, variances, etc.)

  → response function

$$\boxed{R = g_R\left(x_1, x_2, \ldots x_i, \ldots x_n\right)}$$

- Approaches aiming at the calculation of theoretical probability of failure

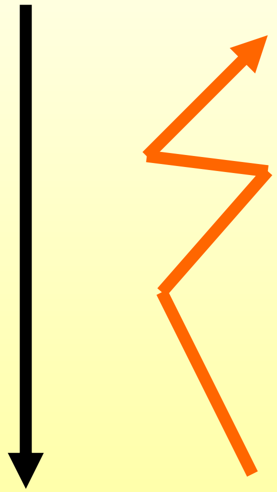
  → limit state function

$$\boxed{Z = g_z\left(x_1, x_2, \ldots x_i, \ldots x_n\right)}$$

$$\boxed{p_f = P\left(Z \leq 0\right)}$$

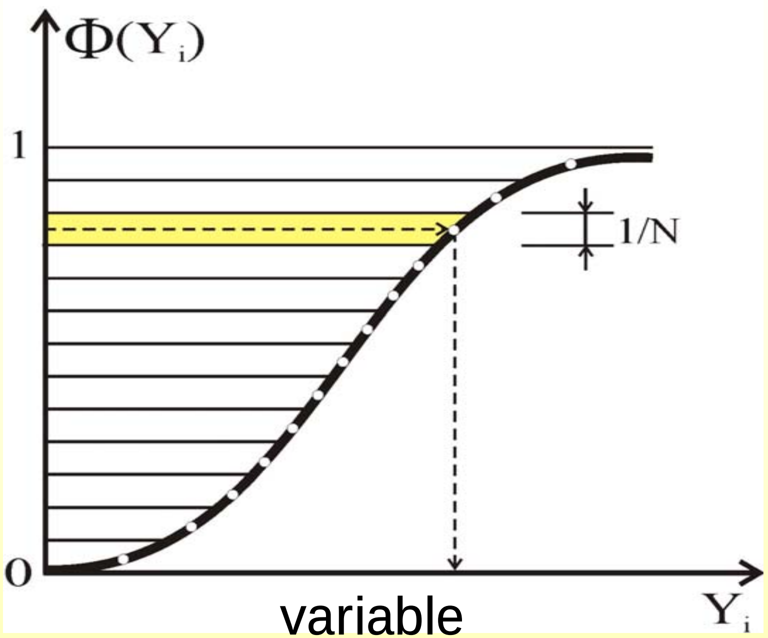
Reliability analysis - computational demands: Number of evaluation of limit state function

- Crude Monte Carlo	1 000 000 000 ….
- Importance sampling	1 000 - 10 000
- Approximation FORM, SORM - design point calculation	100 - 1000
- Response surface	100 - 1000
- Cornell safety index, Curve fitting - MC - LHS	10 - 100



Latin Hypercube Sampling

- The range (0; 1) of PDF $\Phi(Y_i)$ of each random variable Y_i is divided into *N* non-overlapping intervals of equal probability *1/N* (McKay et al. 1979. Iman & Conover 1980, Iman & Shortencarier 1984).
- The centroids are selected randomly based on random permutations of integers.
- Every interval of each variable is used only once during the simulation process.



variable

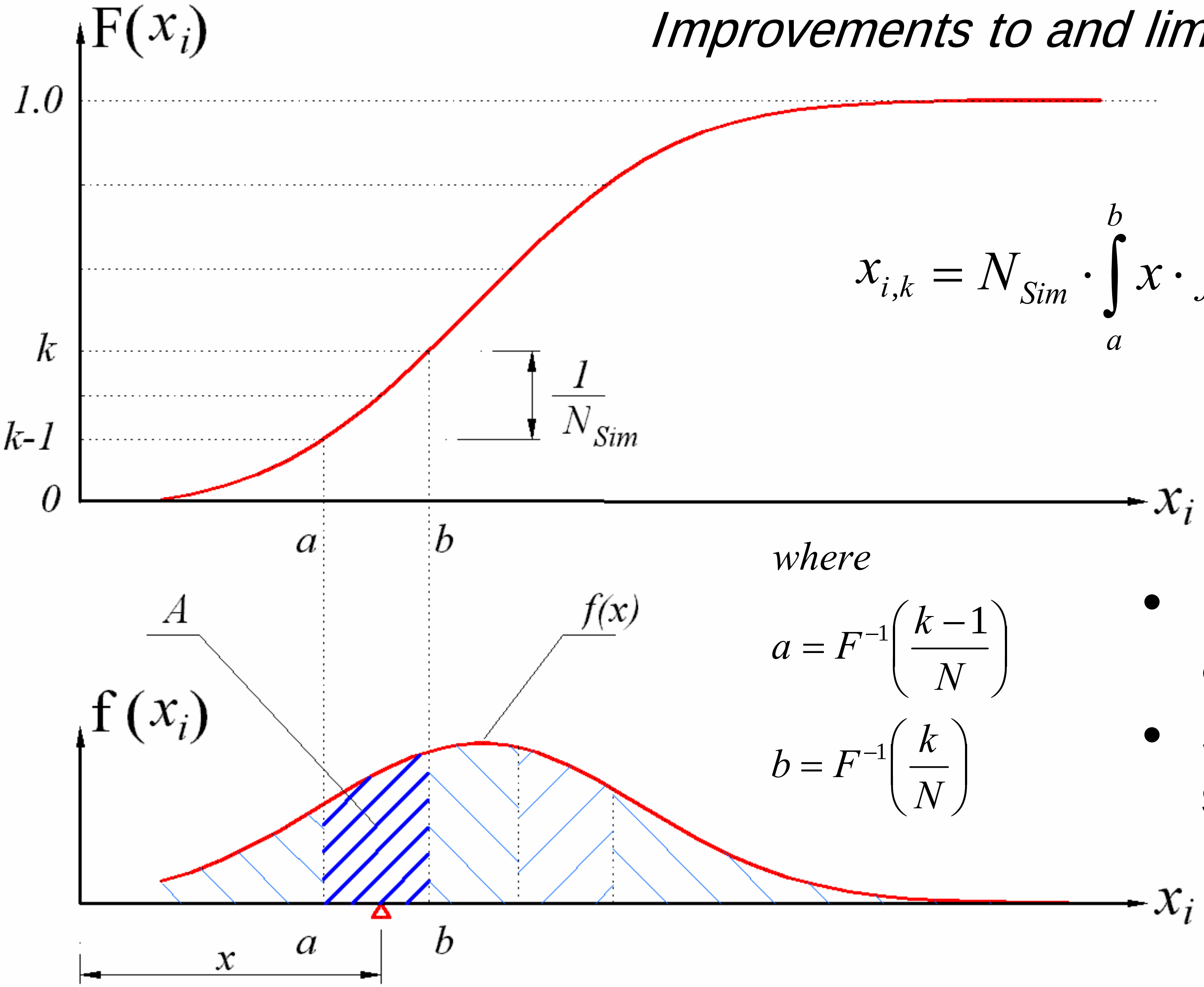
simulation

9	1	10	4	1
4	5	3	7	10
8	3	9	10	8
6	2	8	9	3
10	4	4	8	8
7	10	5	1	2
5	9	6	5	4
2	6	7	2	6
1	7	1	6	7
3	6	2	3	5

LHS: Step 1 - simulation

Huntington & Lyrintzis (1998):

Improvements to and limitations of LHS



$$x_{i,k} = N_{Sim} \cdot \int_a^b x \cdot f\left(x\right) \; dx$$

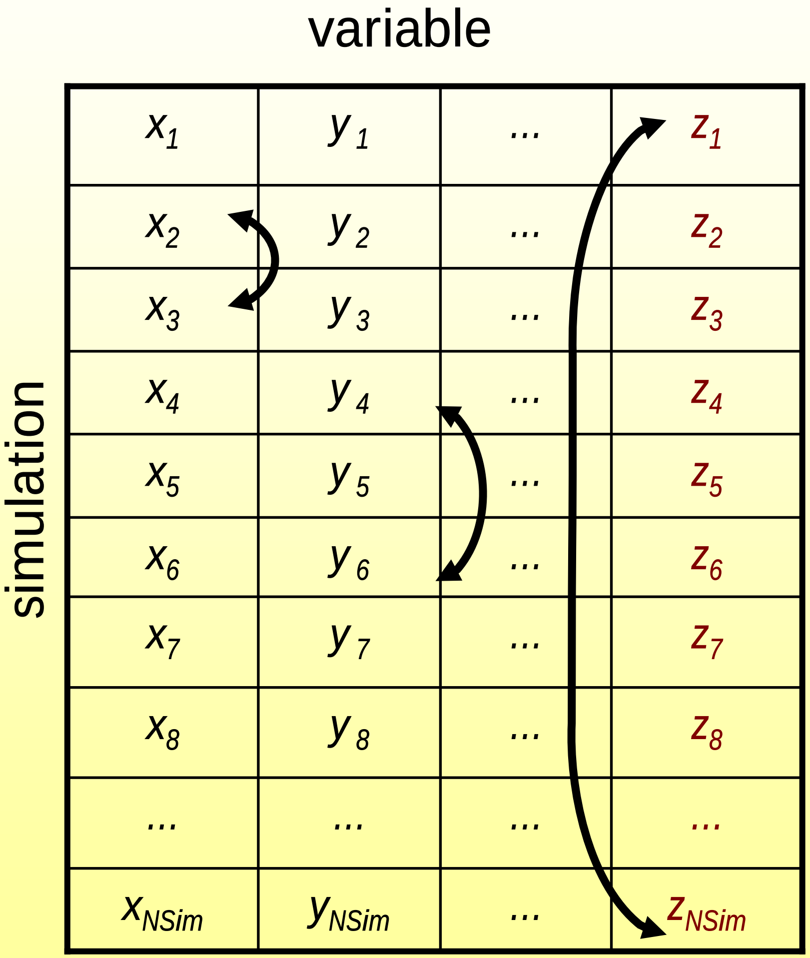
where

$$a = F^{-1}\left(\frac{k-1}{N}\right)$$

$$b = F^{-1}\left(\frac{k}{N}\right)$$

- Mean value: accurately
- Stand. deviation: significant improvement

LHS: Step 2 – imposing statistaical correlation



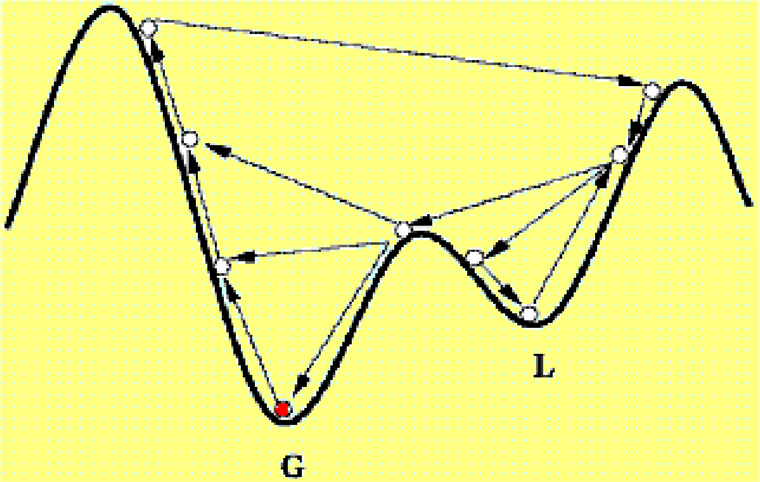
variable

simulation

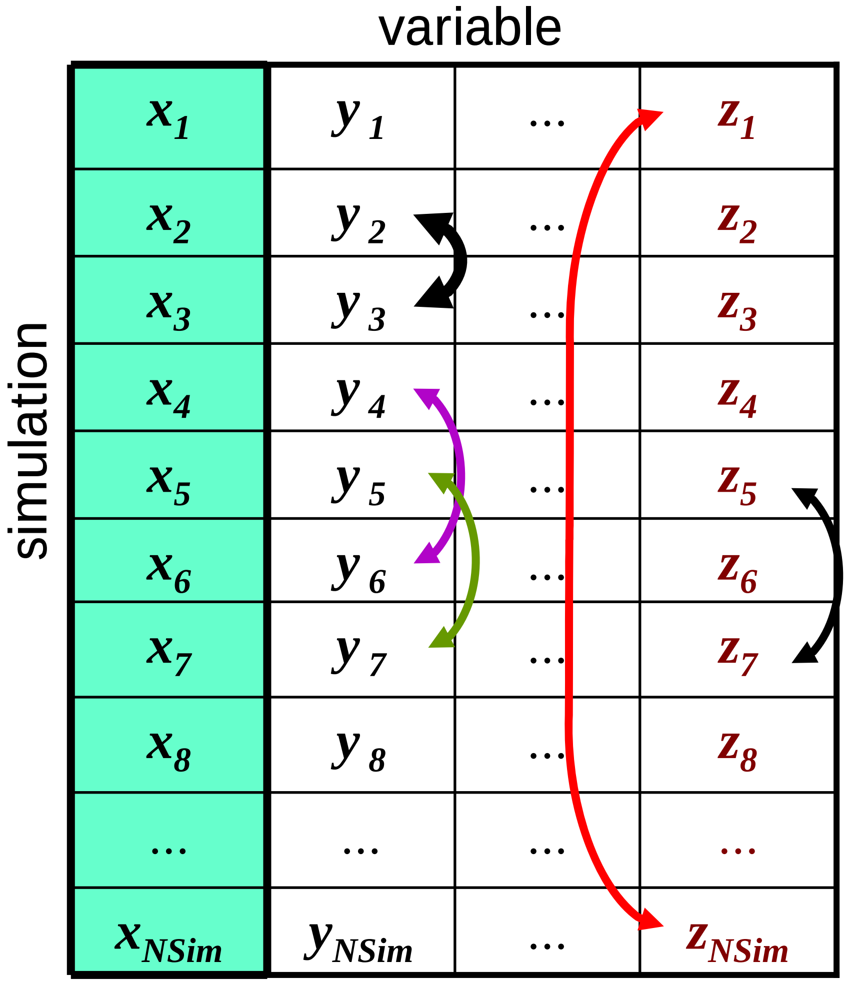
x_1	y_1	...	z_1
x_2	y_2	...	z_2
x_3	y_3	...	z_3
x_4	y_4	...	z_4
x_5	y_5	...	z_5
x_6	y_6	...	z_6
x_7	y_7	...	z_7
x_8	y_8	...	z_8
...	...	...	...
x_{NSim}	y_{NSim}	...	z_{NSim}

- Simulated annealing: Probability to escape from local minima
- Cooling - decreasing of system excitation
- Boltzmann PDF, energetic analogy

$$P_r\left(E\right) \approx e^{\left(\frac{-\Delta E}{k_b \cdot T}\right)}$$



Best ordering (all possible rank combinations). Is it possible to find the *global minimum*?



variable

simulation

x_1	y_1	…	z_1
x_2	y_2	…	z_2
x_3	y_3	…	z_3
x_4	y_4	…	z_4
x_5	y_5	…	z_5
x_6	y_6	…	z_6
x_7	y_7	…	z_7
x_8	y_8	…	z_8
…	…	…	…
x_{NSim}	y_{NSim}	…	z_{NSim}

One column remains stable.

Others permute.

There exist

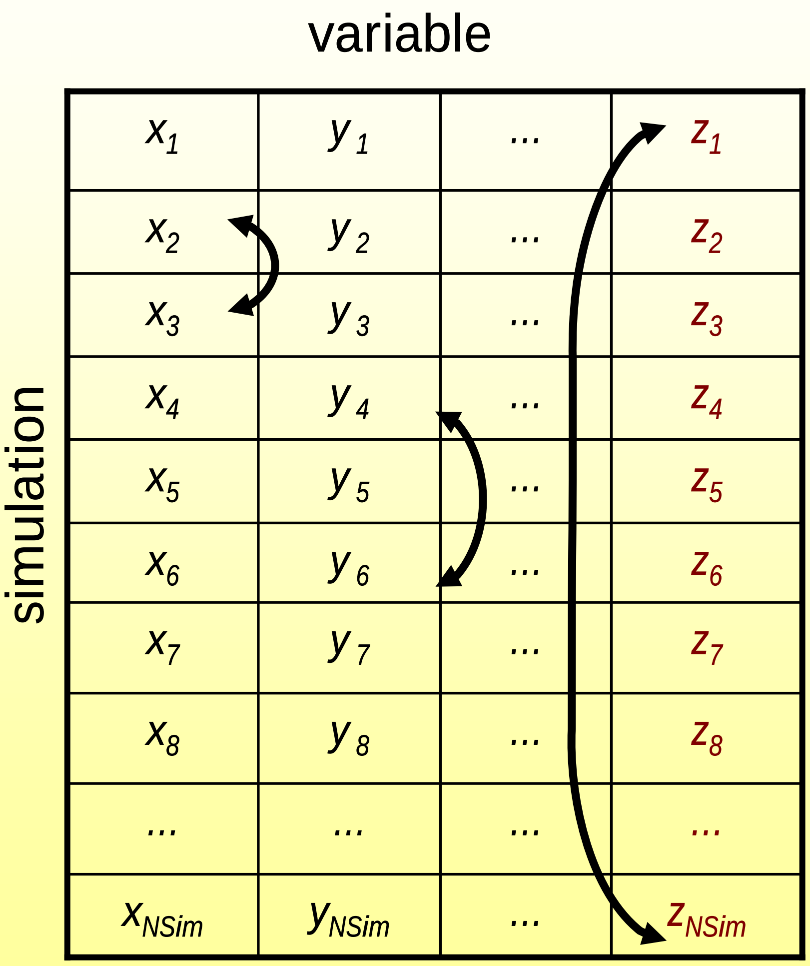
$$\left(N_{Sim}!\right)^{N_{Var}-1}$$ possibilities.

In case of 6 simulations with 5 variables:

$$(6!)^{5-1} = 2.6874\cdot 10^{11}$$

possibilities.

LHS: Step 2 – imposing statistaical correlation



x_1	y_1	…	z_1
x_2	y_2	…	z_2
x_3	y_3	…	z_3
x_4	y_4	…	z_4
x_5	y_5	…	z_5
x_6	y_6	…	z_6
x_7	y_7	…	z_7
x_8	y_8	…	z_8
…	…	…	…
x_{NSim}	y_{NSim}	…	z_{NSim}

- Simulated annealing: Probability to escape from local minima
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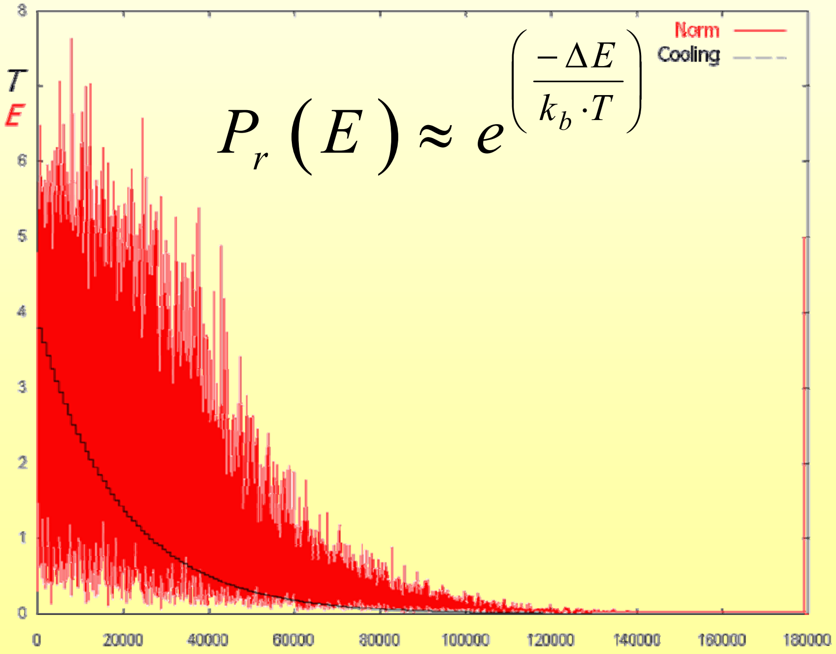
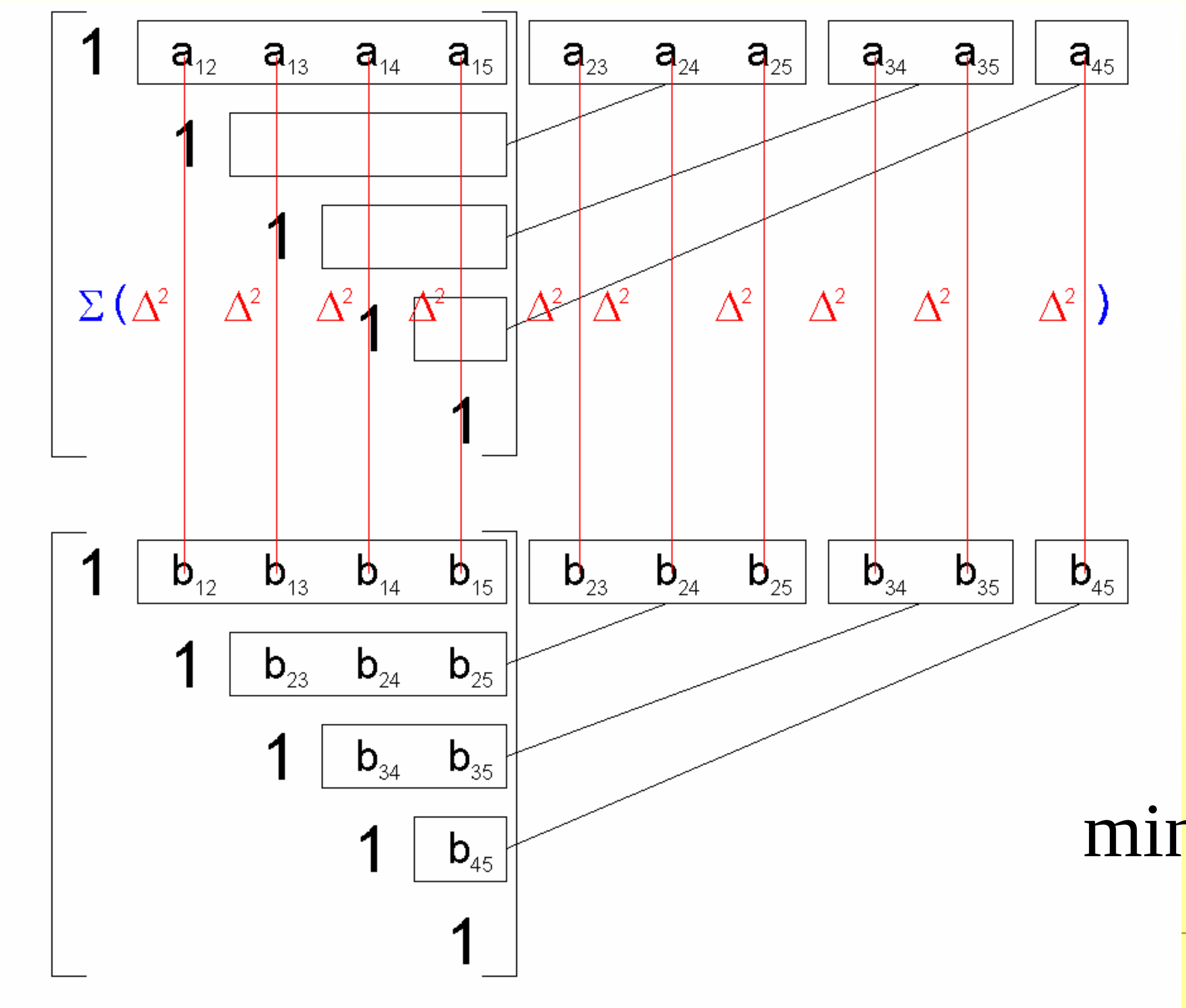


$$P_r\left(E\right) \approx e^{\left(\frac{-\Delta E}{k_b \cdot T}\right)}$$

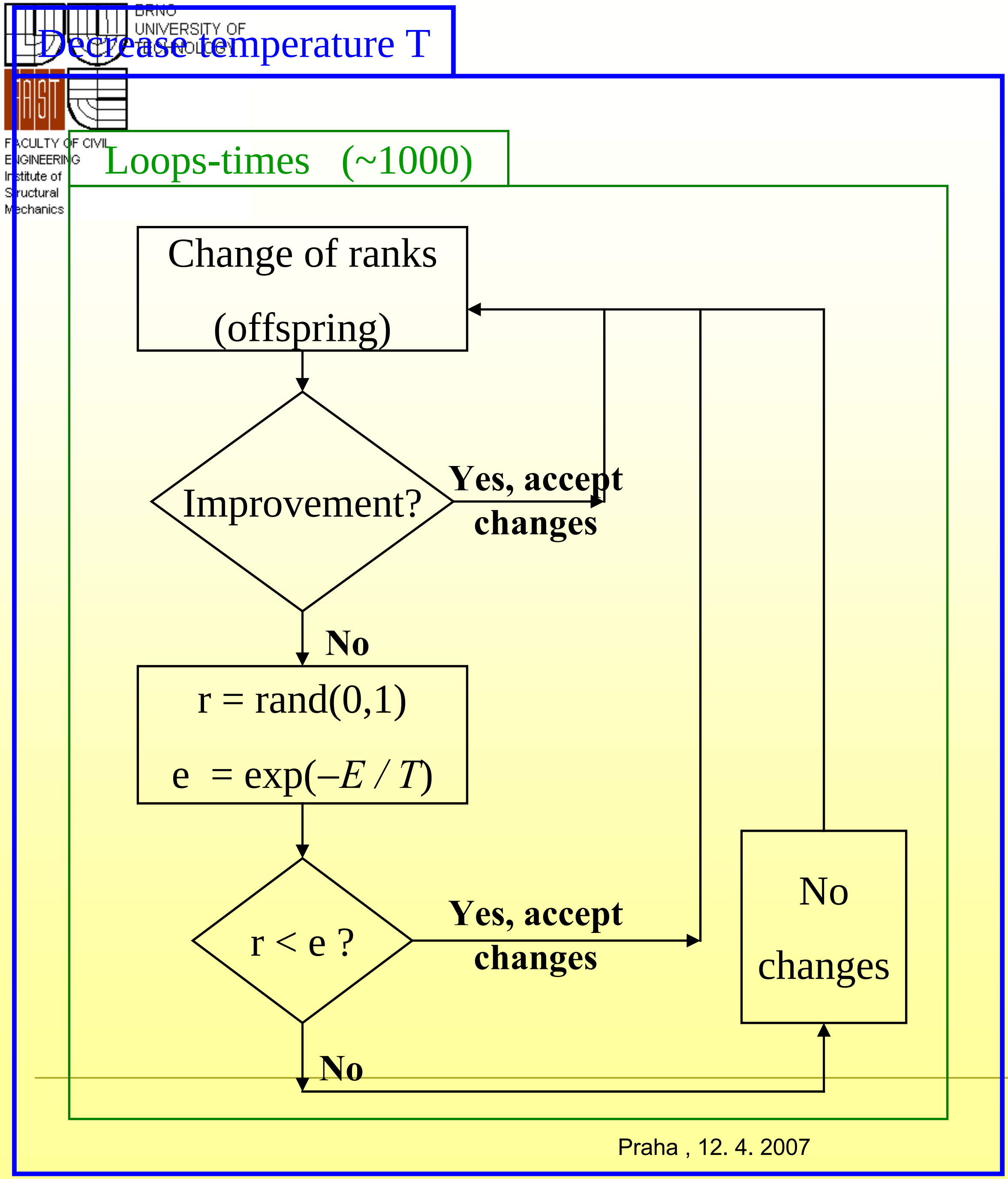
Statistical correlation in LHS - optimization problem



- $a_{i,j}$ - the target correlation matrix
- $b_{i,j}$ - the actual correlation matrix

$$\min \sum_{i,j}^{N,k} \left(a_{i,j} - b_{i,j}\right)^2$$

Simulated annealing



Decrease temperature T

Loops-times (~1000)

Change of ranks (offspring)

Improvement?

Yes, accept changes

No

r = rand(0,1)

e = exp(–*E* / *T*)

r < e ?

Yes, accept changes

No

No changes

$$E_i = Err_i - Err_{i-1}$$

$$k_b = 1$$

$$P_r(E) \approx e^{\left(\frac{-E}{k_b \cdot T}\right)}$$

Numerical test 1:
diminish spurious correlation

$$\boldsymbol{K}= \begin{array}{|c|c|c|c|c|} \hline 1 & & & & \\ \hline 0 & 1 & & & \\ \hline 0 & 0 & 1 & & \\ \hline 0 & 0 & 0 & 1 & \\ \hline 0 & 0 & 0 & 0 & 1 \\ \hline \end{array}$$

- 5 variables and 6 simulations.

1. ULHS, iterations (Spearman)
2. Simulated annealing, (PC 400MHz *3 sec*)

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Diminish spurious correlation comparison

Cholesky decomp. iterative ULHS (Spearman)

$$E^2_{overall} = 0.22$$

1.4080	0.6867	0.2142	1.4080	-0.2142
-1.4080	-0.6867	1.4080	0.2142	0.2142
-0.6867	0.2142	-0.6867	0.6867	-1.4080
0.2142	-1.4080	-1.4080	-0.2142	0.6867
0.6867	-0.2142	0.6867	-1.4080	-0.6867
-0.2142	1.4080	-0.2142	-0.6867	1.4080

1	0.21	-0.20	0.07	-0.05
	1	0.11	0.12	0.11
		1	-0.08	-0.10
			1	-0.27
				1

target correlation matrix

Samples:

1.4080
0.6867
0.2142
-0.2142
-0.6867
-1.4080

Difference:

|.|<0.1
|.|<0.2
|.|<0.3

Proposed algorithm (Simulated Annealing)

$$E^2_{overall} = 0.04$$

0.2142	1.4080	0.6867	1.4080	0.2142
-1.4080	-0.6867	-0.2142	0.6867	-0.6867
0.6867	-1.4080	0.2142	0.2142	1.4080
-0.2142	0.2142	1.4080	-1.4080	-0.2142
1.4080	-0.2142	-0.6867	-0.2142	-1.4080
-0.6867	0.6867	-1.4080	-0.6867	0.6867

1	-0.10	0.06	-0.01	-0.08
	1	0.06	0.09	-0.09
		1	0	0.06
			1	0.05
				1

target correlation matrix

Numerical test– imposition of target statistical correlation

$$\boldsymbol{K}=\begin{array}{|c|c|c|c|c|}\hline 1 & & & & \\ \hline 0.2 & 1 & & & \\ \hline 0.2 & 0.6 & 1 & & \\ \hline 0.2 & 0.6 & 0.6 & 1 & \\ \hline 0.2 & 0.5 & 0.2 & 0.5 & 1 \\ \hline \end{array}$$

- 5 variables and 6 simulations.
  Number of simulation - great influence.
- All possibilities *50 minutes*

- Simulated annealing *3 sec* (always finds local minima)

Simulated annealing - results

Starting sampling matrix

$$E^2_{overall} = 3.79$$

1.4080	1.4080	1.4080	1.4080	1.4080
0.6867	0.6867	0.6867	0.6867	0.6867
0.2142	0.2142	0.2142	0.2142	0.2142
-0.2142	-0.2142	-0.2142	-0.2142	-0.2142
-0.6867	-0.6867	-0.6867	-0.6867	-0.6867
-1.4080	-1.4080	-1.4080	-1.4080	-1.4080

1	1.0	1.0	1.0	1.0
0.2	1	1.0	1.0	1.0
0.2	0.6	1	1.0	1.0
0.2	0.6	0.6	1	1.0
0.2	0.5	0.2	0.5	1

Samples:

1.4080
0.6867
0.2142
-0.2142
-0.6867
-1.4080

Difference:

|.|<0.1
|.|<0.2
|.|<0.3

Proposed genetic algorithm (Simulated Annealing)

$$E^2_{overall} = 0.00\ 4$$

1.4080	1.4080	1.4080	1.4080	1.4080
0.2142	-0.2142	-0.2142	-1.4080	-0.6867
-0.6867	-0.6867	0.6867	0.2142	-1.4080
-1.4080	0.2142	-0.6867	-1.4080	-0.2142
1.4080	0.6867	0.2142	0.6867	0.6867
0.6867	-1.4080	-1.4080	-0.6867	0.2142

1	0.165	0.237	0.203	0.517
0.2	1	0.590	0.588	0.537
0.2	0.6	1	0.615	0.193
0.2	0.6	0.6	1	0.503
0.2	0.5	0.2	0.5	1

LHS: Simulated annealing – weighted

variable

variable

1	*0,9*	*0,9*
	1	*-0,9*
		1

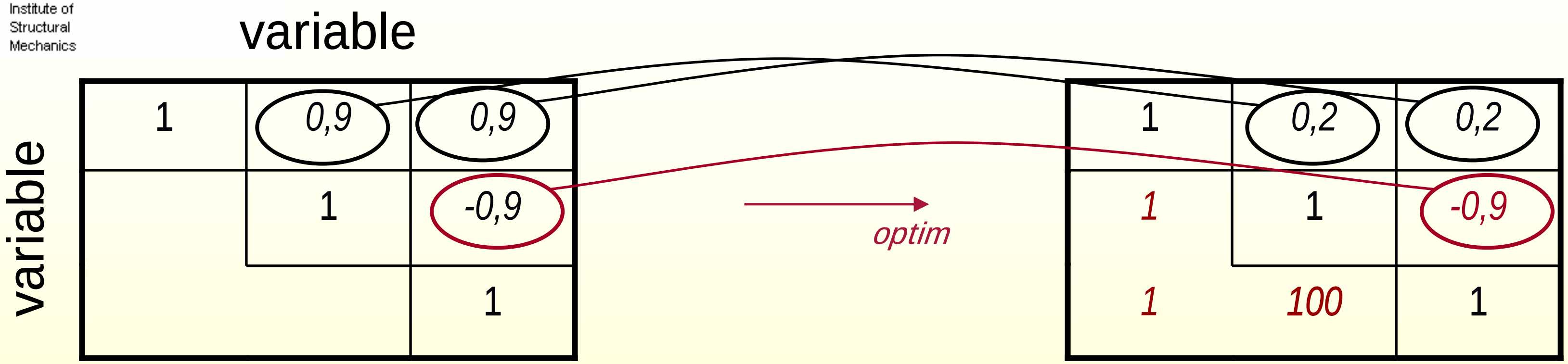
$$\xrightarrow[\text{optim}]{}$$

1	*0,49*	*0,49*
	1	*-0,49*
		1

- Resulting correlation matrix is positive definite and error is uniformly distributed among all coefficients - compromise

☹ Positive definiteness of ***K***

LHS: Simulated annealing – weighted



1	*0,9*	*0,9*
	1	*-0,9*
		1

1	*0,2*	*0,2*
1	1	*-0,9*
1	*100*	1

- Resulting correlation matrix is positive definite and error is uniformly distributed among all coefficients
- Weighted method: suppression of selected coefficients

☺ Positive definiteness of ***K***

Simulation of random fields

- Essential topic in stochastic continuum mechanics.
- The need for accurate representation and simulation in SFEM.
- Various methods ...
- Orthogonal transformation of covariance matrix
  (Schuëller et al. 1990, Liu et al. 1995)
  ➡ Small number of random variables to represent random fields.

- Latin Hypercube Sampling (LHS)
  Small number of simulations.
  ➡ Combination: A new alternative method

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Simulation of random fields

Orthogonal transformation of covariance matrix and LHS

$$R_{aa}(\xi) = \sigma_0^2 \exp\left[-\left(\frac{|\xi|}{d}\right)^2\right]$$

$$\mathbf{C}_{XX} = \mathbf{\Phi \Lambda \Phi}^{\mathrm{T}}$$

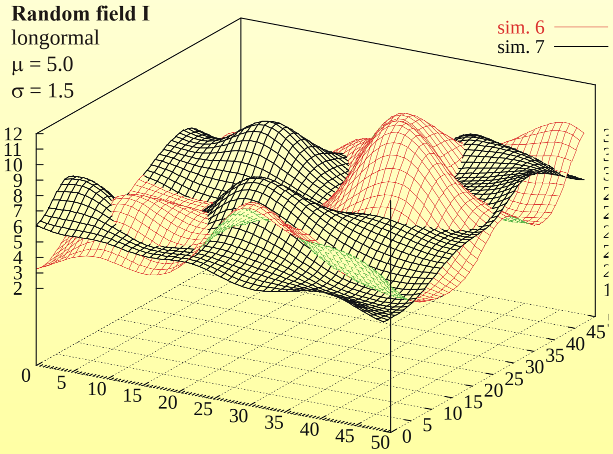
$$\mathbf{C}_{YY} = \mathbf{\Lambda}$$

Φ - eigenvector matrix

Λ - Cov. matrix in uncorrelated space (diagonal) : eigenvalues $\lambda_1,\lambda_2,\ldots,\lambda_{nd}$

Simulation - *uncorrelated*
Gaussian random variables:

$Y^T = [Y_1, Y_{2,}\ldots, Y_{nr}]$: $\mathbf{X} = \mathbf{\Phi Y}$

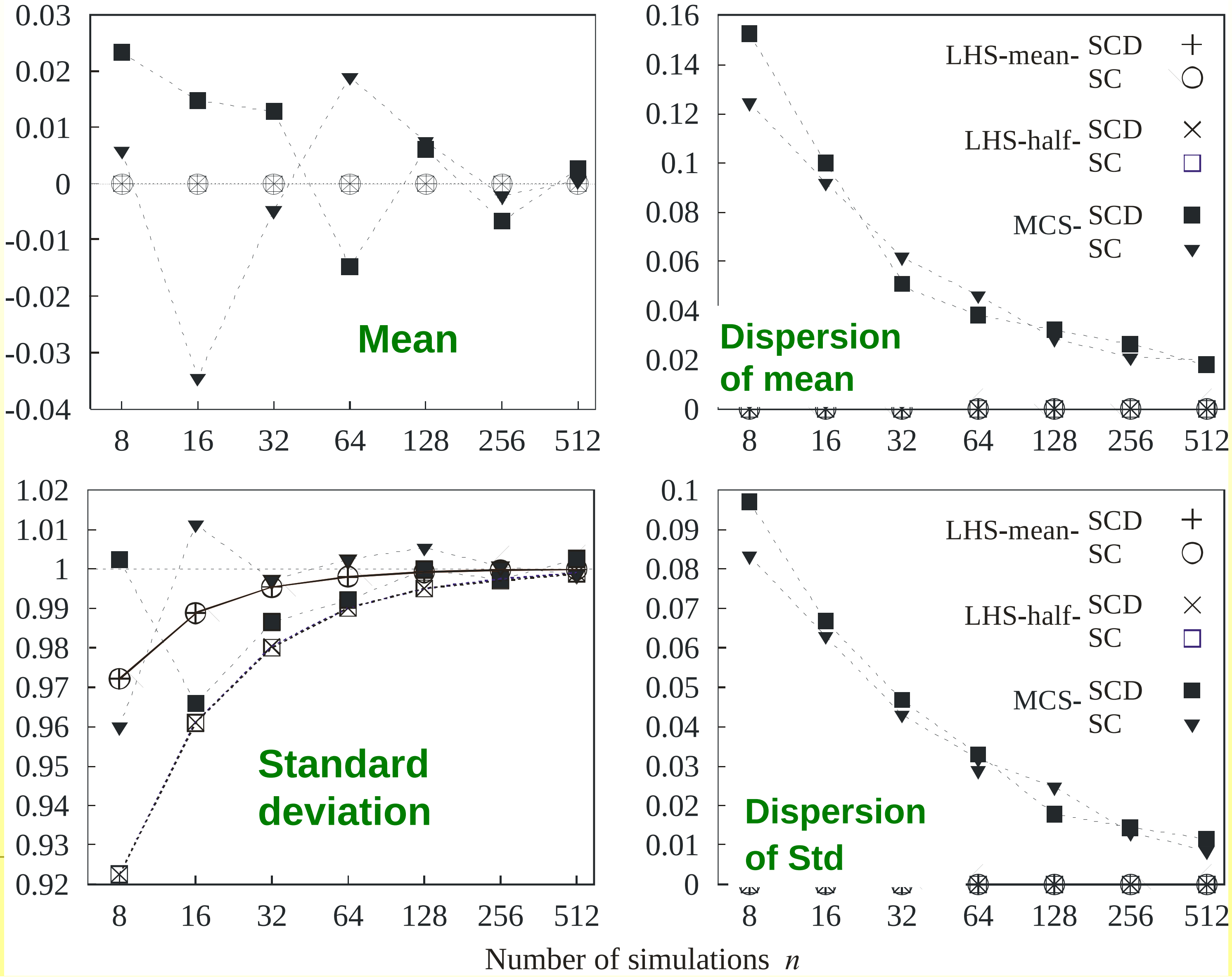


Y — MCS
Y — LHS

?

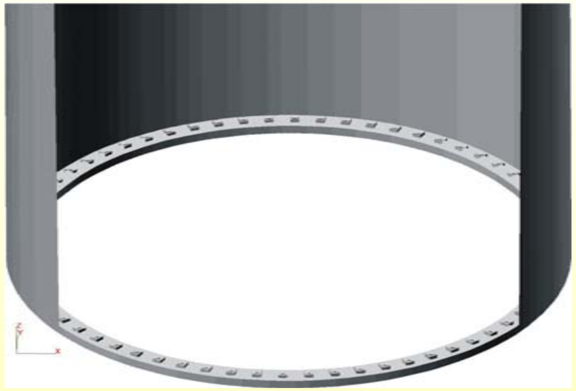
Vořechovský, Novák - Icossar 2005

Comparison of convergence to target fields statistics



Novák, D., Lawanwisut, W., Bucher C. (2000). Simulation of random field based on orthogonal transformation of covariance matrix and Latin Hypercube Sampling, MC 2000, Monte Carlo.

Bucher, C. and Ebert, M. (2000) Load Carrying Behavior of Prestressed Bolted Steel Flanges Considering Random Geometrical Imperfections, *PMC2000, University of Notre Dame, USA.*



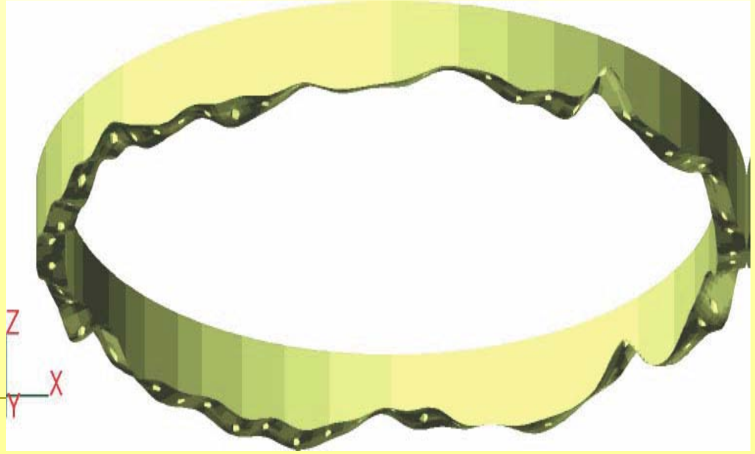
Finite element model of flange

- SFEM model with 13000 DOF
- random field to describe geometrical imperfections
- 1500 random variables

$$1500 \xrightarrow{\text{reduction}} 128$$

Method	Mean Value [MNm]	Coeff. of Variation [%]
LHS (32 samples)	22.3	0.087
MCS (200 samples)	21.9	0.076

Statistics of ultimate bending moment



Realization of random field

Sensitivity analysis

Nonparametric rank-order correlation between input variables ane output response variable

- Kendall tau $\tau_i = \tau\left(q_{ji}, p_j\right), \quad j = 1, 2, \ldots, N$
- Spearman

$$r^s = 1 - \frac{6 \sum_{i=1}^{n} d_i^2}{n(n-1)(n+1)}$$

- Robust - uses only orders
- Additional result of LHS simulation, no extra effort
- Bigger correlation coefficient = high sensitivity
- Relative measure of sensitivity (-1, 1)

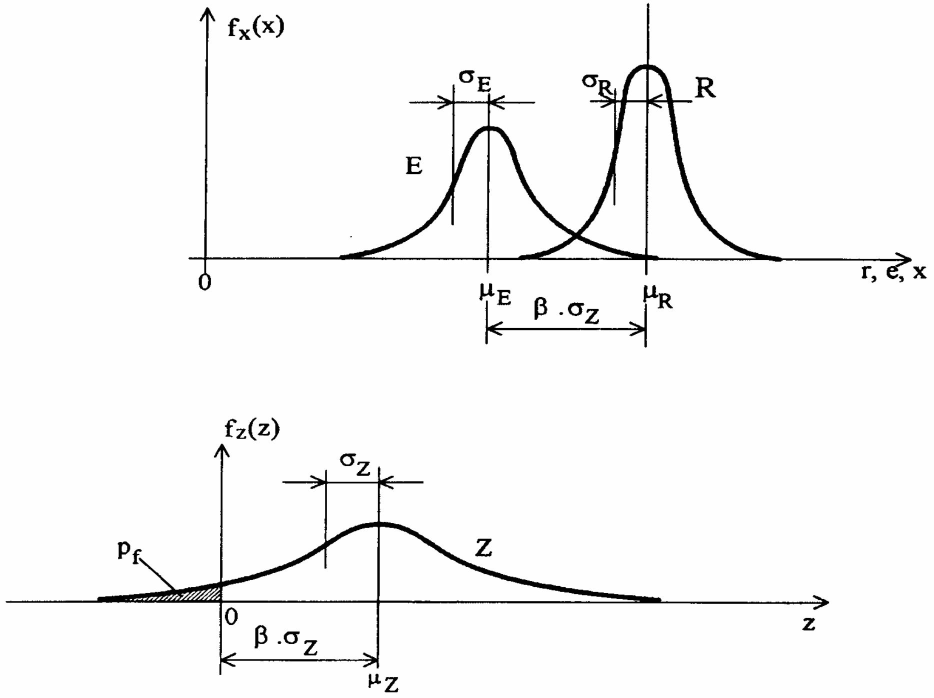
INPUT	OUTPUT
$x_{1,1}$	R_1
…	…
…	…
…	…
$x_{1,N}$	$R_{,N}$

↓

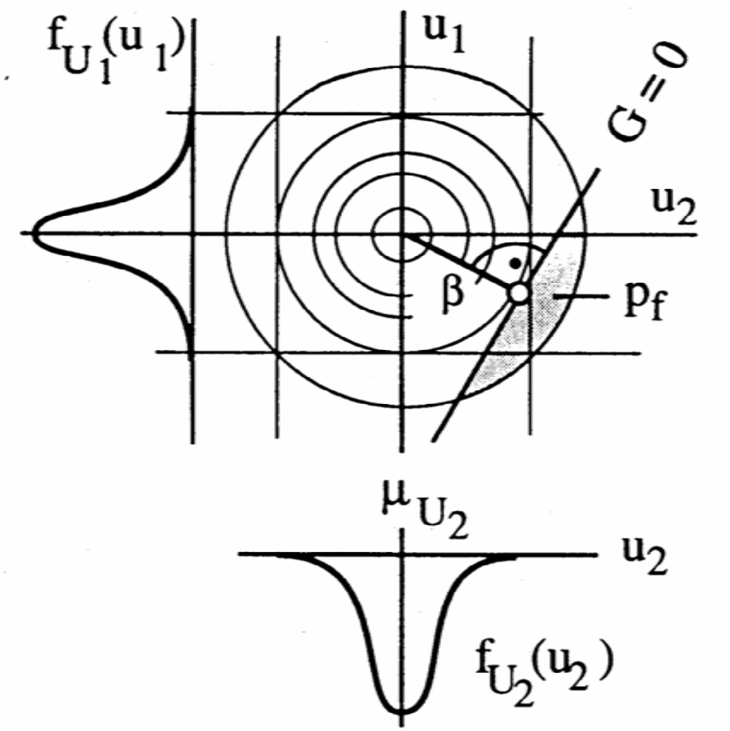
INPUT	OUTPUT
$q_{1,1}$	p_1
…	…
…	…
…	…
$q_{1,N}$	p_N

Reliability analysis

- Simplified – rough estimates, as constrained by extremally small number of simulations (10-100)!
- Cornell safety index $\beta = \dfrac{\mu_Z}{\sigma_Z}$
- Curve fitting
- FORM, importance sampling response surface…



The method of Hasofer and Lind, 1974



- Hasofer and Lind, 1974 - important step
- Transformation of the limit state function into so-called standard space

$$U_1 = \frac{R-\mu_R}{\sigma_R} \qquad U_2 = \frac{S-\mu_S}{\sigma_S}$$

- New variables with mean value 0 and standard deviation 1
- In the new coordinate system the line G=R-S no longer passes through origin
- HL safety index - the distance from the design point to origin
- correct in case of normally distributed variables, for non-normally - a good approximation

$$\beta = \sqrt{u^T u}$$

Subject to $g(X) = 0$,

$$p_f = \Phi(-\beta)$$

$$G = R - S = (U_1 \cdot \sigma_R + \mu_R) - (U_2 \cdot \sigma_S + \mu_S)$$

Identification of material parameters

Numerical model of structure

↓

appropriate material model

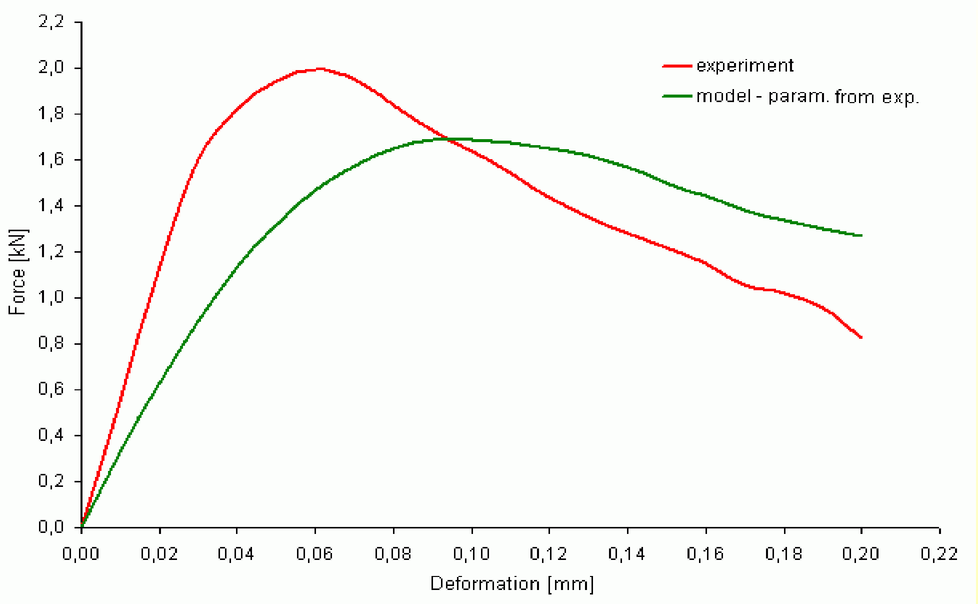
many material parameters

Information about parameters →

- experimental data
- recommended formulas
- engineering estimation

Identification of material parameters

Primary calculation



Correction of parameters:

- „trial – and – error“ method
- sofisticated identification methods
  - artificial neural network + stochastic calculations (LHS)

Artificial neural network

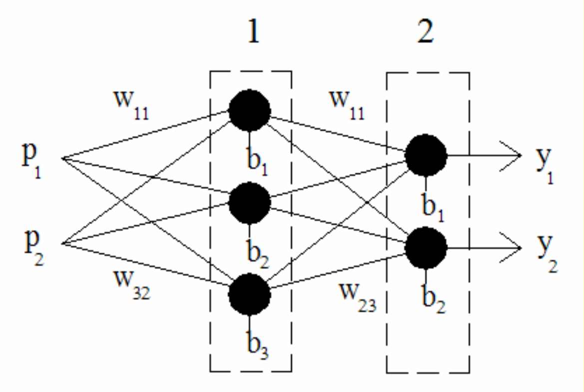
Modeling of processes in brain

(1943 - McCulloch-Pitts Perceptron)

Various fields of technical practice

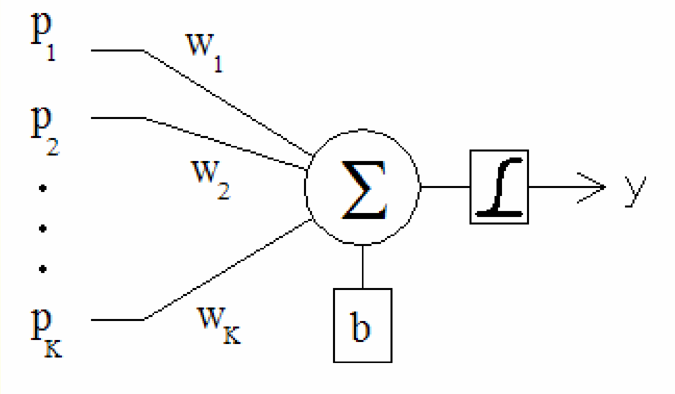
Neural network type – Multi-layer perceptron:

- set of neurons arranged in several layers
- all neurons in one layer are connected
  with all neurons of the following layer



Artificial neural network

NEURON:



Output from 1 neuron:

$$\boxed{y = f(x) = f\left(\sum_k (w_k \cdot p_k) + b\right)}$$

k – number of input impuls (1,...,K)
w_k – weight coefficient of connecting path from *k*-th neuron of previous layer
p_k – impuls from *k*-th neuron previous layer
b – bias of neuronu
f – transfer function of neuron

Artificial neural network

Two phases: → active period (simulation of process)
→ adaptive period (training)

Training of network:

- training set, i.e. ordered pair [p_i, y_i]

input and output vector

Minimization of criterion:

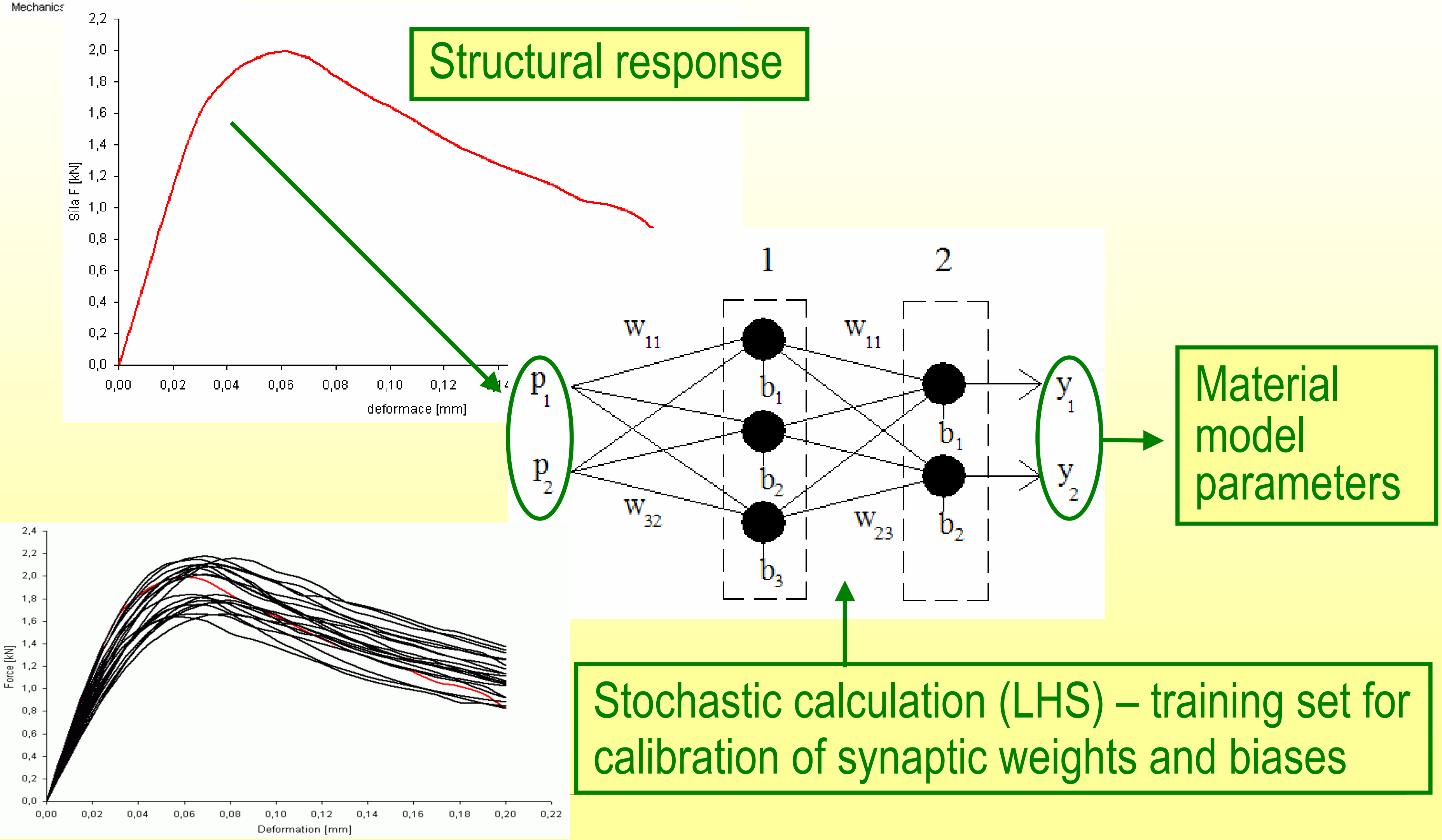
$$\boxed{E = \frac{1}{2}\sum_{i=1}^{N}\sum_{k=1}^{K}\left(y_{ik}^{v} - y_{ik}^{*}\right)^{2}}$$

N – number of ordered pairs input - output in training set;

y_{ik}^{*} – required output value of k-th output neuron at i-th input;

y_{ik}^{v} – real output value (at same input).

Principle of identification method



Structural response

Material model parameters

Stochastic calculation (LHS) – training set for calibration of synaptic weights and biases

FReET software development

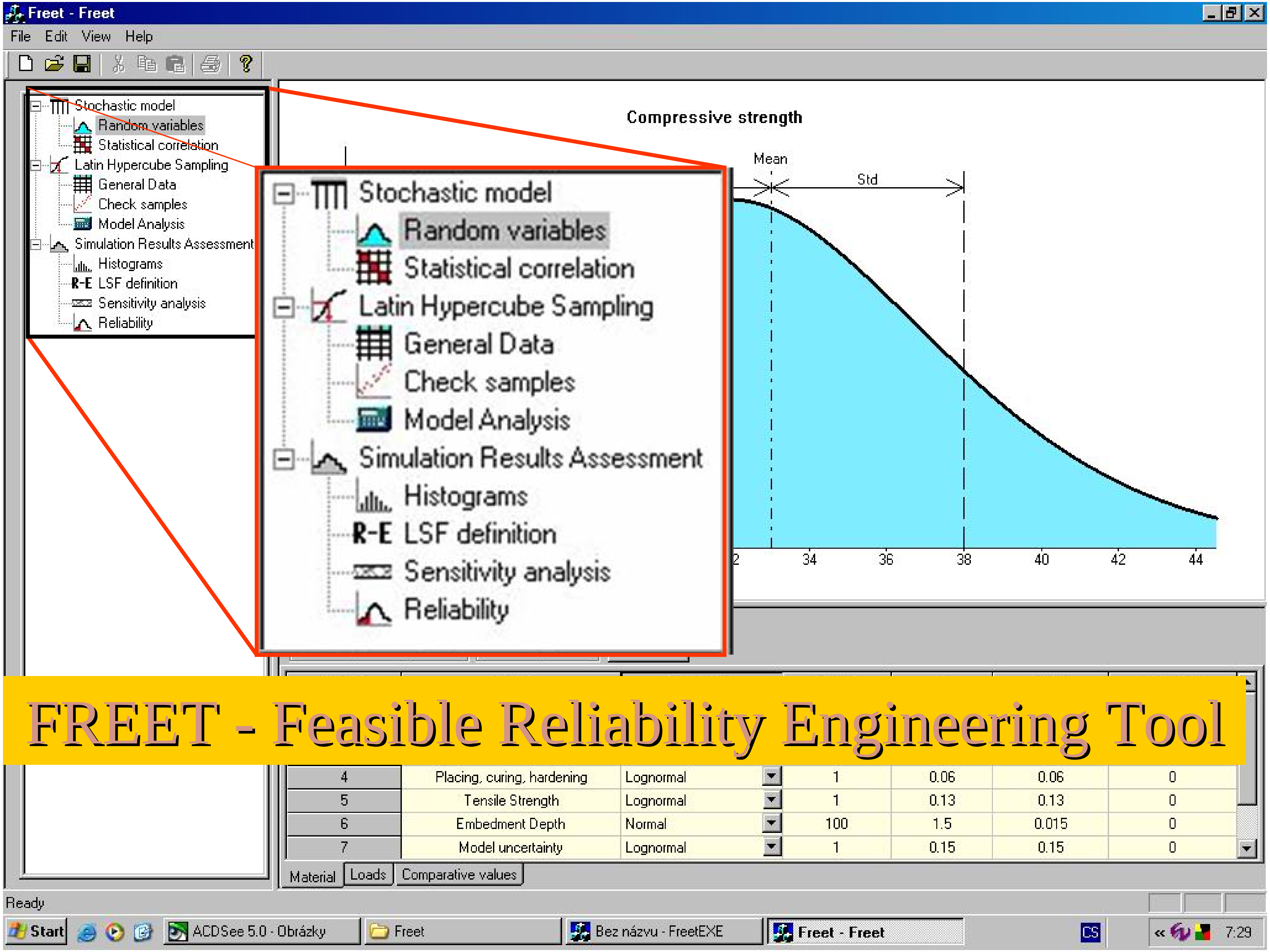
- Stand alone module - definition of reliability problem (user-defined limit state/response function) in programming language (C++, FORTRAN) – DLL function or by equation interpretor

- Integration with software ATENA - nonlinear fracture mechanics of reinforced concrete structures (Červenka Consulting) – SARA software shell

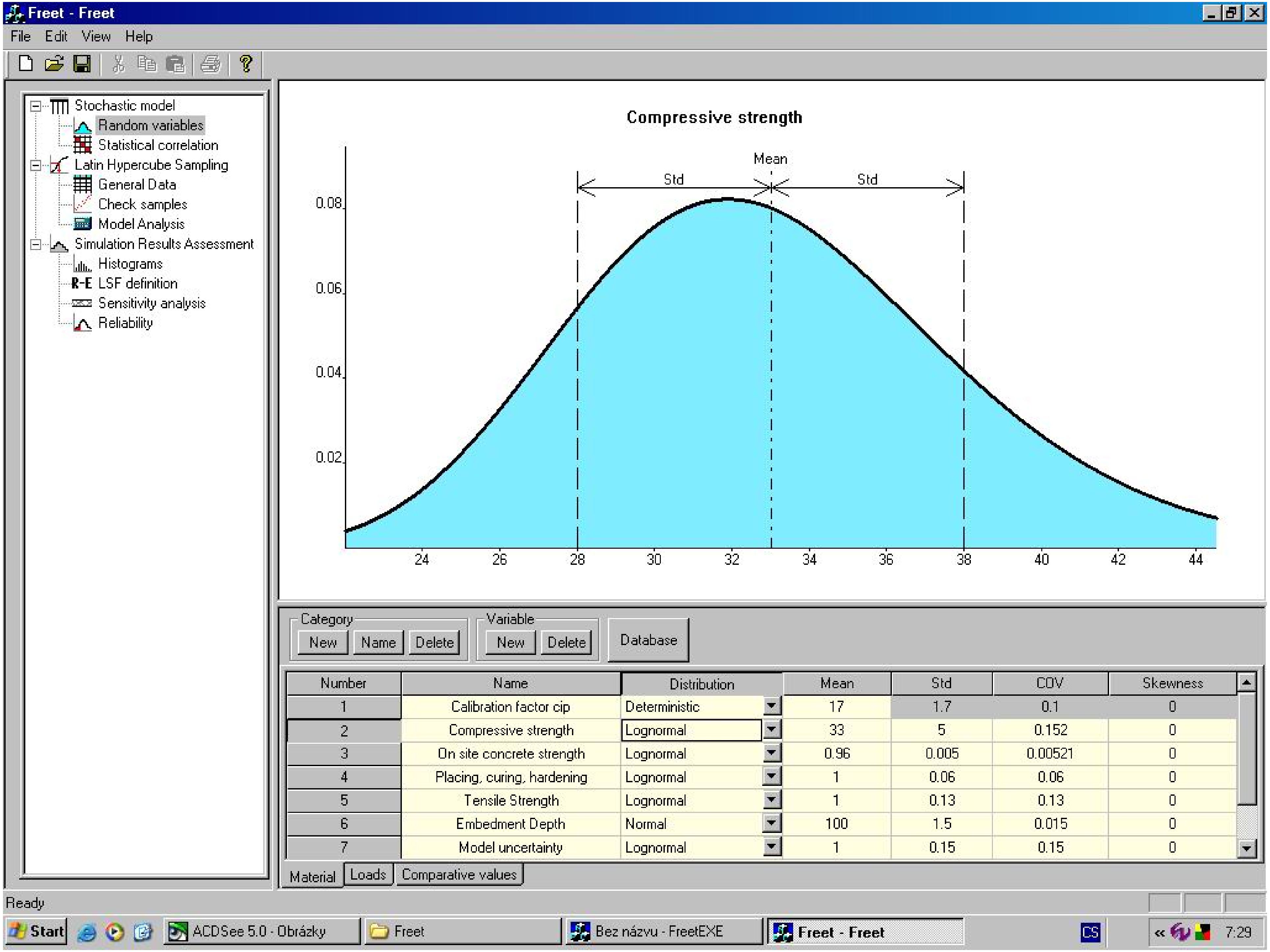
Software Freet

- Freet version 1.1 – December 2004
  - Enhanced set of PDF
  - a possibility to add new comparative values without need to perform simulation
  - outputs organization and printing possibilities
  - USB hasp
- Freet version 1.2 – February 2005
  - Net version (BOKU computer lab installation)
  - 1 Hasp for SARA, ATENA, FREET
  - sensitivity graphical output enhanced (also what-if-studies, parametric study)
- Freet version 1.3 – June 2005
  - Weighting for correlation matrix input
  - Response Surface basics
  - File->New clearing results and input
  - Graphics enhancement and checking
  - Random fields basics
- Freet version 1.4 – May 2006, new features:
  - Graphics enhancement and checking
  - Random fields implementation – verified
  - Possibility to define a parameter for easy parametric study with graphical output
  - New type of probabilty distribution: Bounded normal PDF
  - Automatic running of FREET from command line
- Freet version 1.5 – January/February 2007, new features:
  - More general interface to third-parties programs – now DLL and BAT, EXE files communication via text input/output files
  - FORM – First Order Reliability Methods

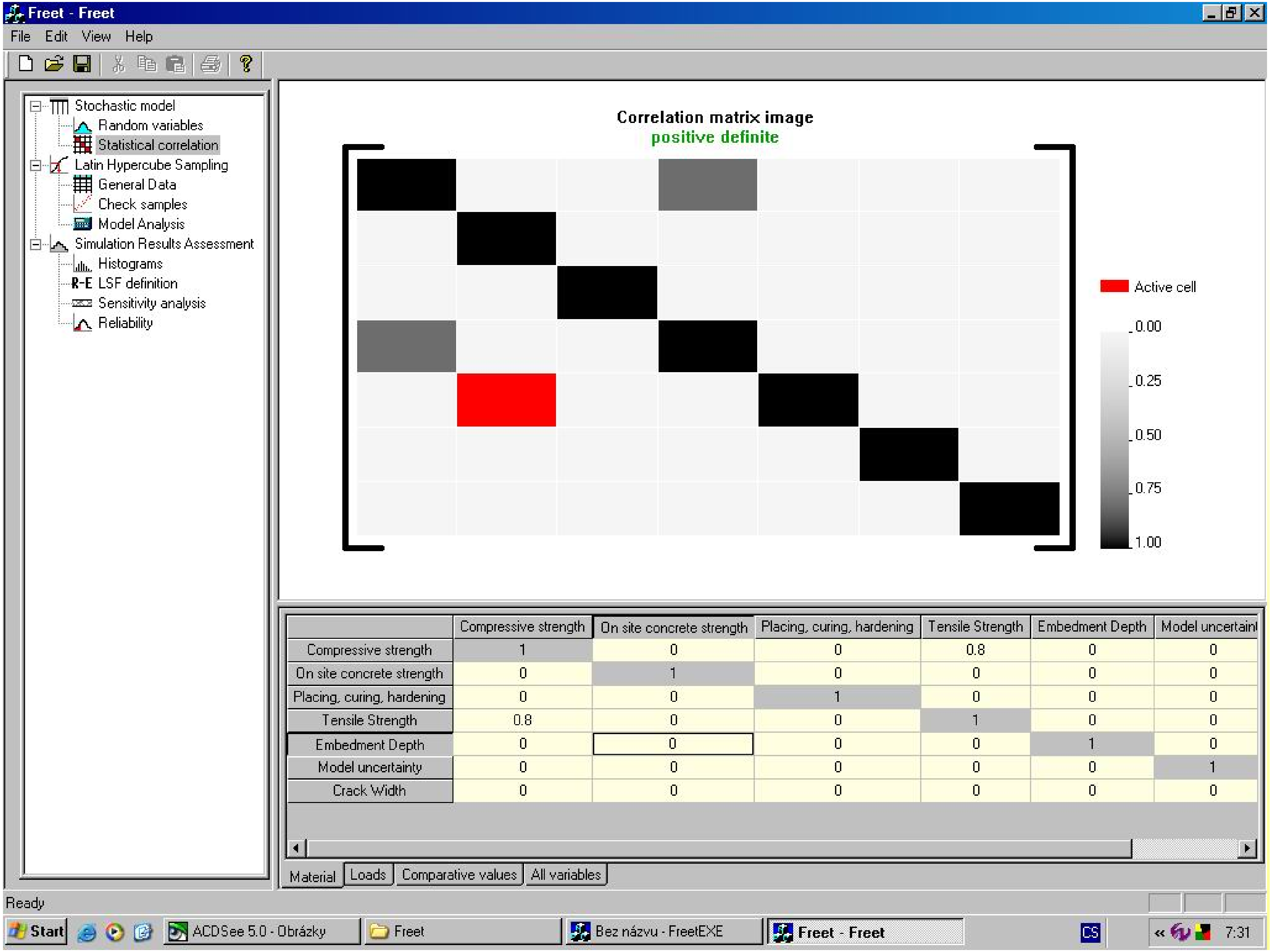
FREET - Feasible Reliability Engineering Tool



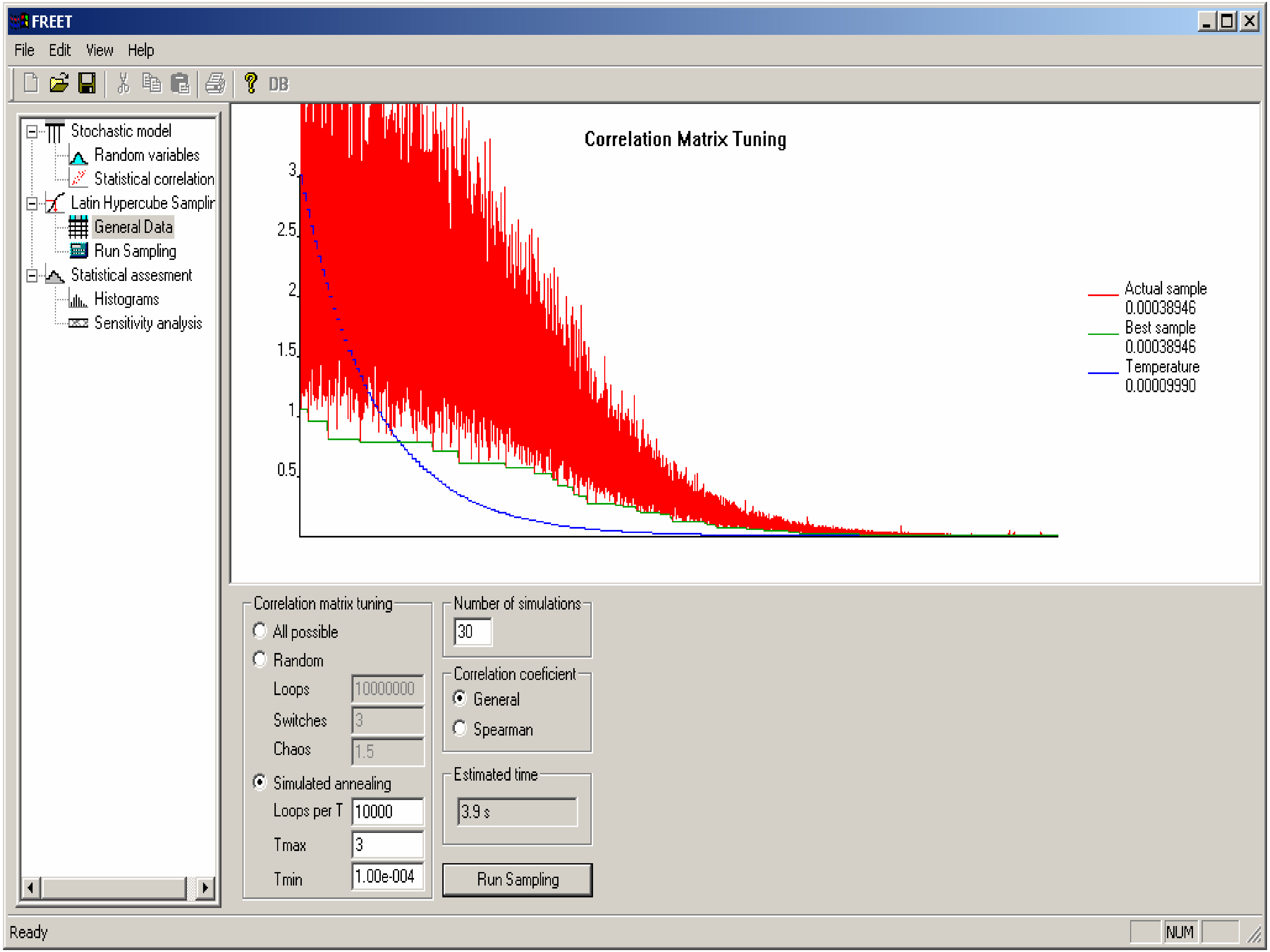
4	Placing, curing, hardening	Lognormal	1	0.06	0.06	0
5	Tensile Strength	Lognormal	1	0.13	0.13	0
6	Embedment Depth	Normal	100	1.5	0.015	0
7	Model uncertainty	Lognormal	1	0.15	0.15	0



Number	Name	Distribution	Mean	Std	COV	Skewness
1	Calibration factor cip	Deterministic	17	1.7	0.1	0
2	Compressive strength	Lognormal	33	5	0.152	0
3	On site concrete strength	Lognormal	0.96	0.005	0.00521	0
4	Placing, curing, hardening	Lognormal	1	0.06	0.06	0
5	Tensile Strength	Lognormal	1	0.13	0.13	0
6	Embedment Depth	Normal	100	1.5	0.015	0
7	Model uncertainty	Lognormal	1	0.15	0.15	0



	Compressive strength	On site concrete strength	Placing, curing, hardening	Tensile Strength	Embedment Depth	Model uncertain
Compressive strength	1	0	0	0.8	0	0
On site concrete strength	0	1	0	0	0	0
Placing, curing, hardening	0	0	1	0	0	0
Tensile Strength	0.8	0	0	1	0	0
Embedment Depth	0	0	0	0	1	0
Model uncertainty	0	0	0	0	0	1
Crack Width	0	0	0	0	0	0



Correlation matrix tuning

○ All possible

○ Random

Loops 10000000

Switches 3

Chaos 1.5

● Simulated annealing

Loops per T 10000

Tmax 3

Tmin 1.00e-004

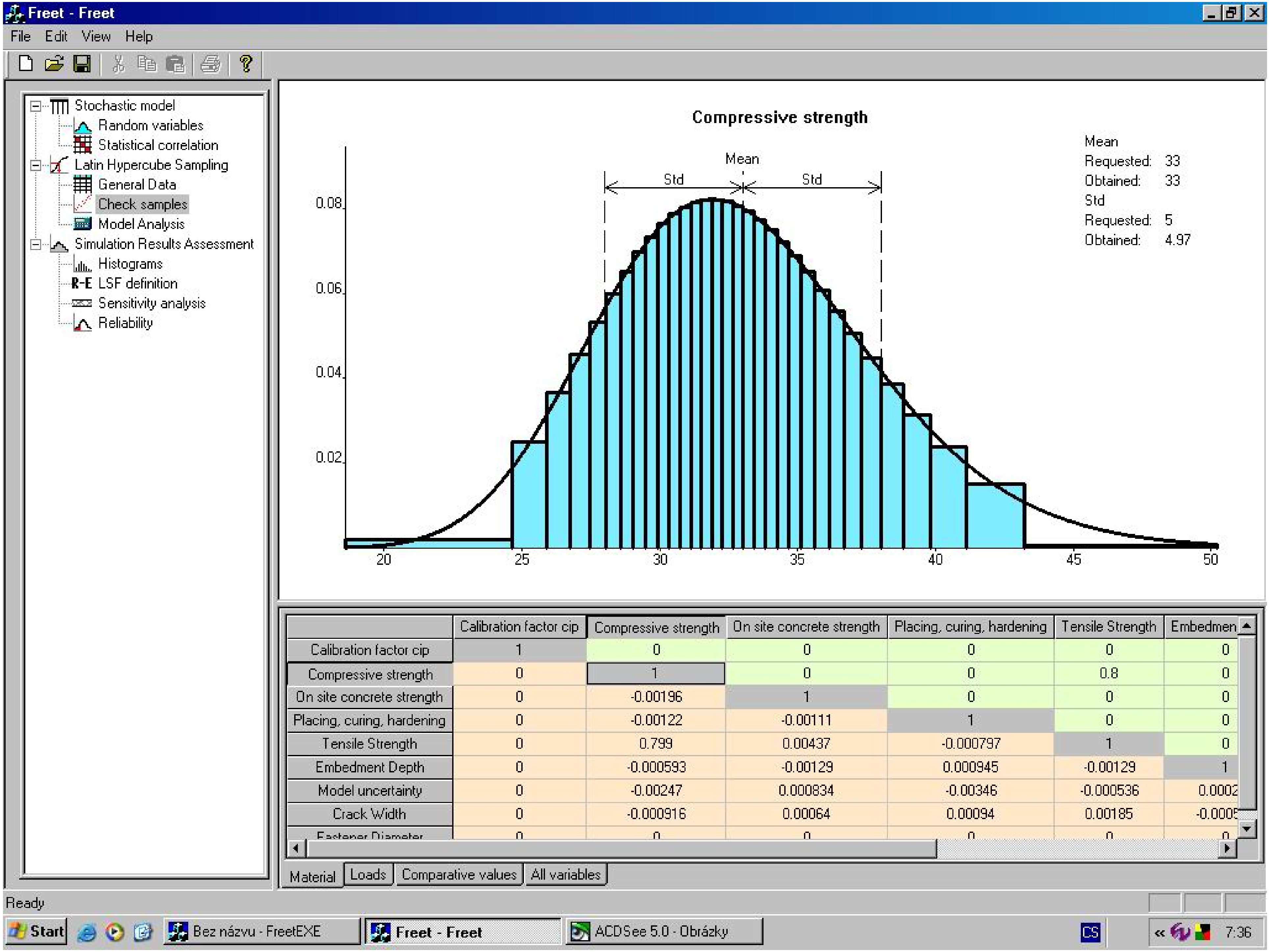
Number of simulations 30

Correlation coeficient

● General

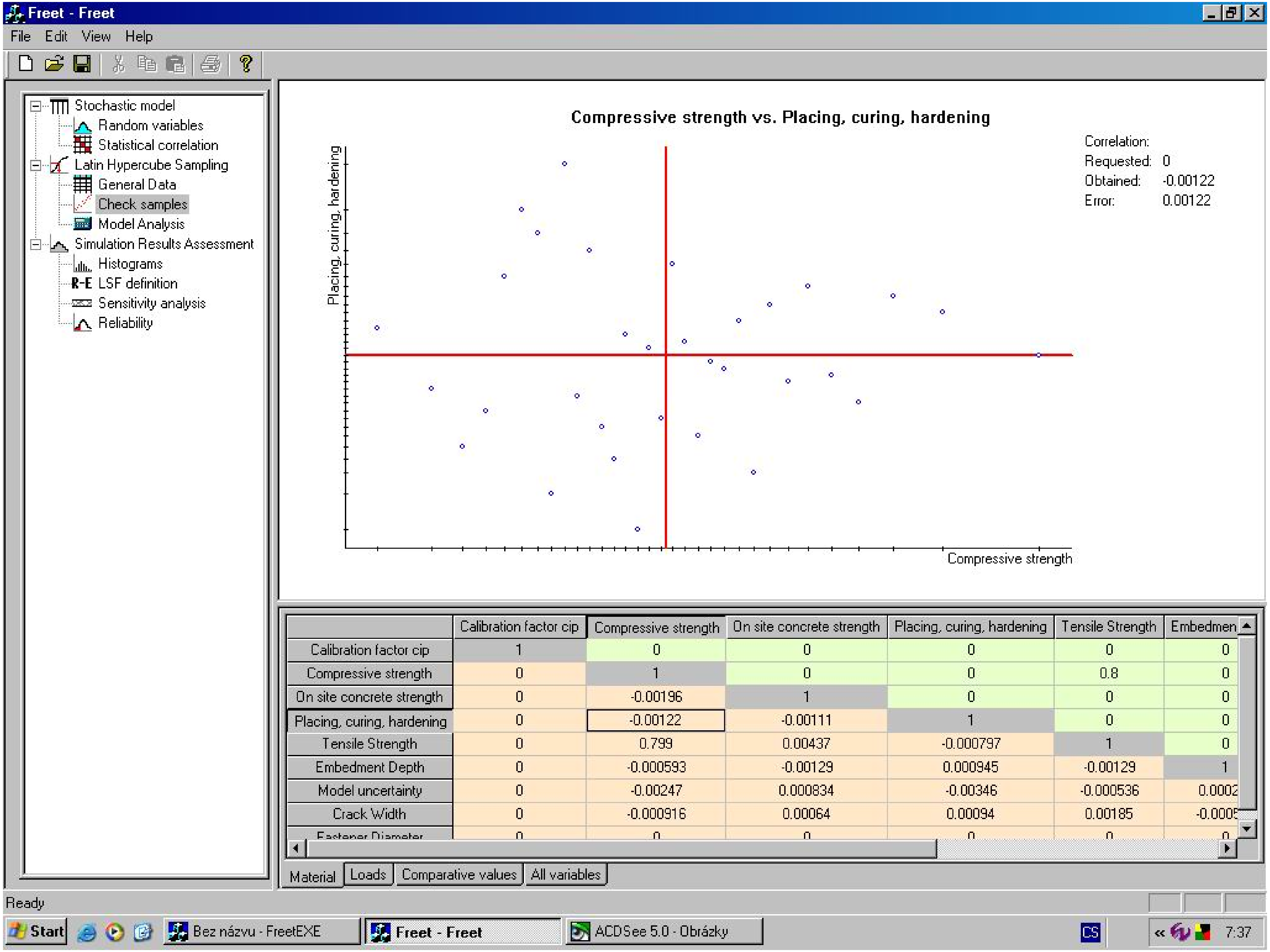
○ Spearman

Estimated time 3.9 s



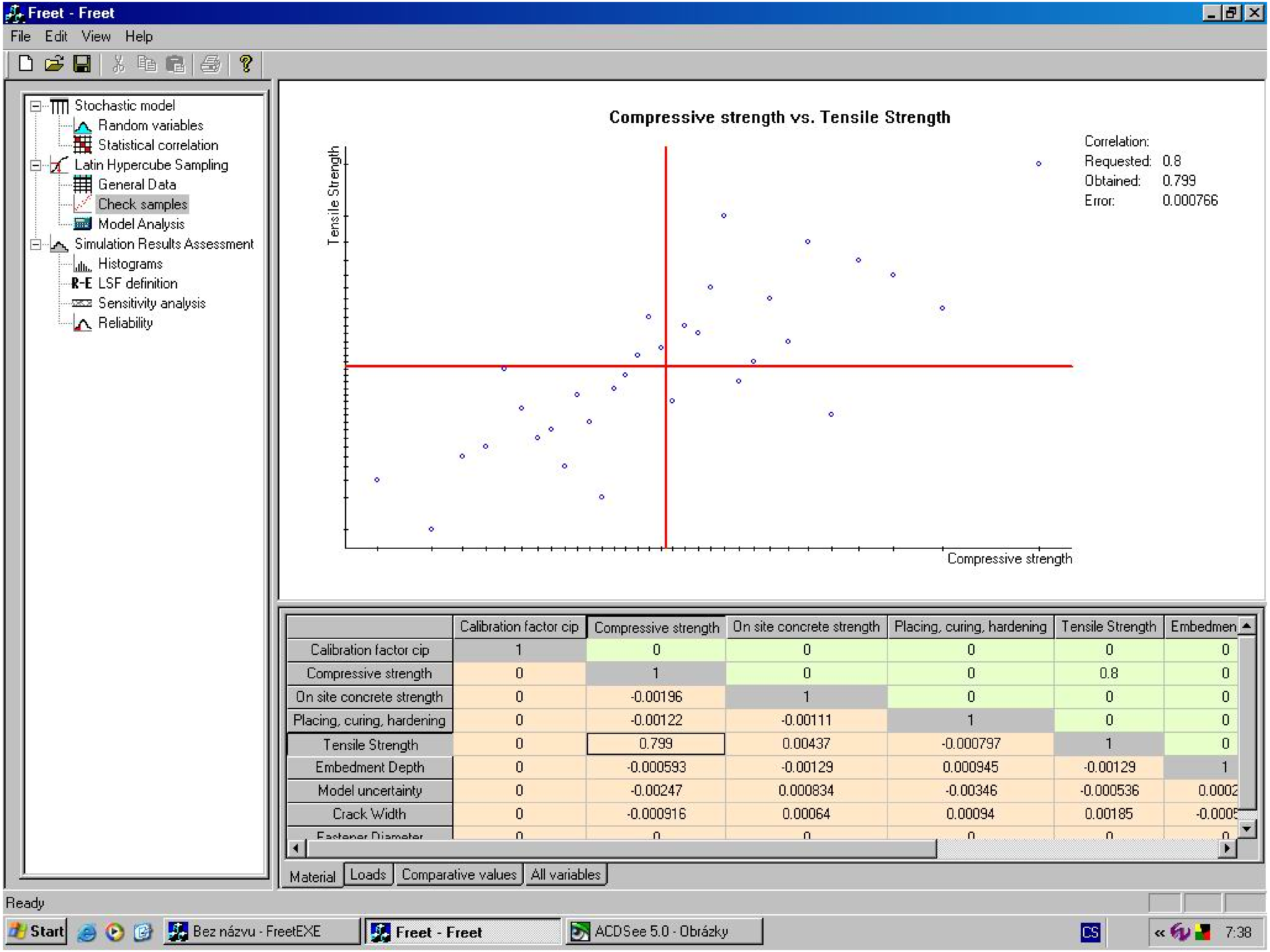
Mean
Requested: 33
Obtained: 33
Std
Requested: 5
Obtained: 4.97

	Calibration factor cip	Compressive strength	On site concrete strength	Placing, curing, hardening	Tensile Strength	Embedmen
Calibration factor cip	1	0	0	0	0	0
Compressive strength	0	1	0	0	0.8	0
On site concrete strength	0	-0.00196	1	0	0	0
Placing, curing, hardening	0	-0.00122	-0.00111	1	0	0
Tensile Strength	0	0.799	0.00437	-0.000797	1	0
Embedment Depth	0	-0.000593	-0.00129	0.000945	-0.00129	1
Model uncertainty	0	-0.00247	0.000834	-0.00346	-0.000536	0.0002
Crack Width	0	-0.000916	0.00064	0.00094	0.00185	-0.0005
Fastener Diameter	0	0	0	0	0	0



Correlation:
Requested: 0
Obtained: -0.00122
Error: 0.00122

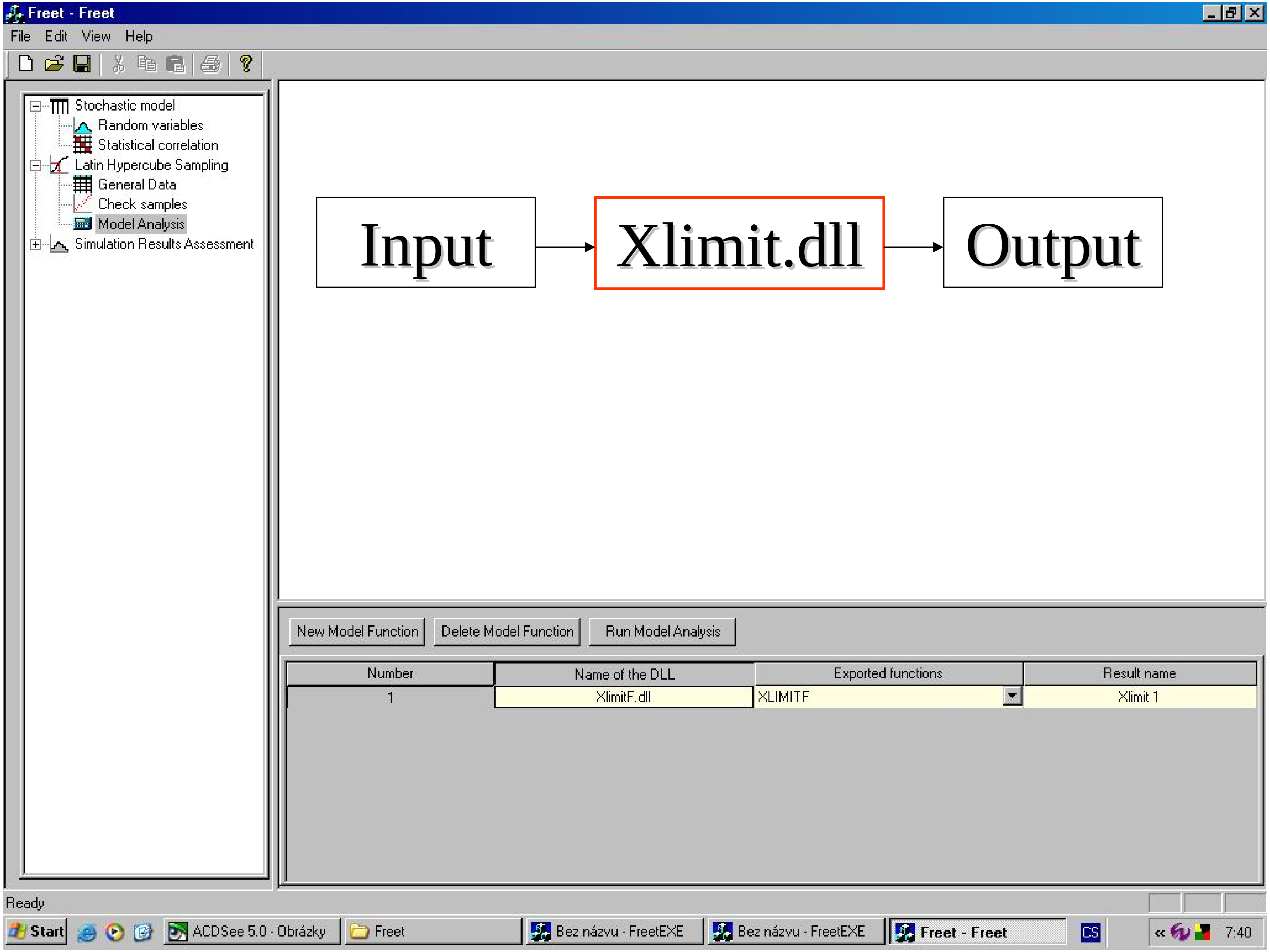
	Calibration factor cip	Compressive strength	On site concrete strength	Placing, curing, hardening	Tensile Strength	Embedmen
Calibration factor cip	1	0	0	0	0	0
Compressive strength	0	1	0	0	0.8	0
On site concrete strength	0	-0.00196	1	0	0	0
Placing, curing, hardening	0	-0.00122	-0.00111	1	0	0
Tensile Strength	0	0.799	0.00437	-0.000797	1	0
Embedment Depth	0	-0.000593	-0.00129	0.000945	-0.00129	1
Model uncertainty	0	-0.00247	0.000834	-0.00346	-0.000536	0.0002
Crack Width	0	-0.000916	0.00064	0.00094	0.00185	-0.0005
Fastener Diameter	0	0	0	0	0	0



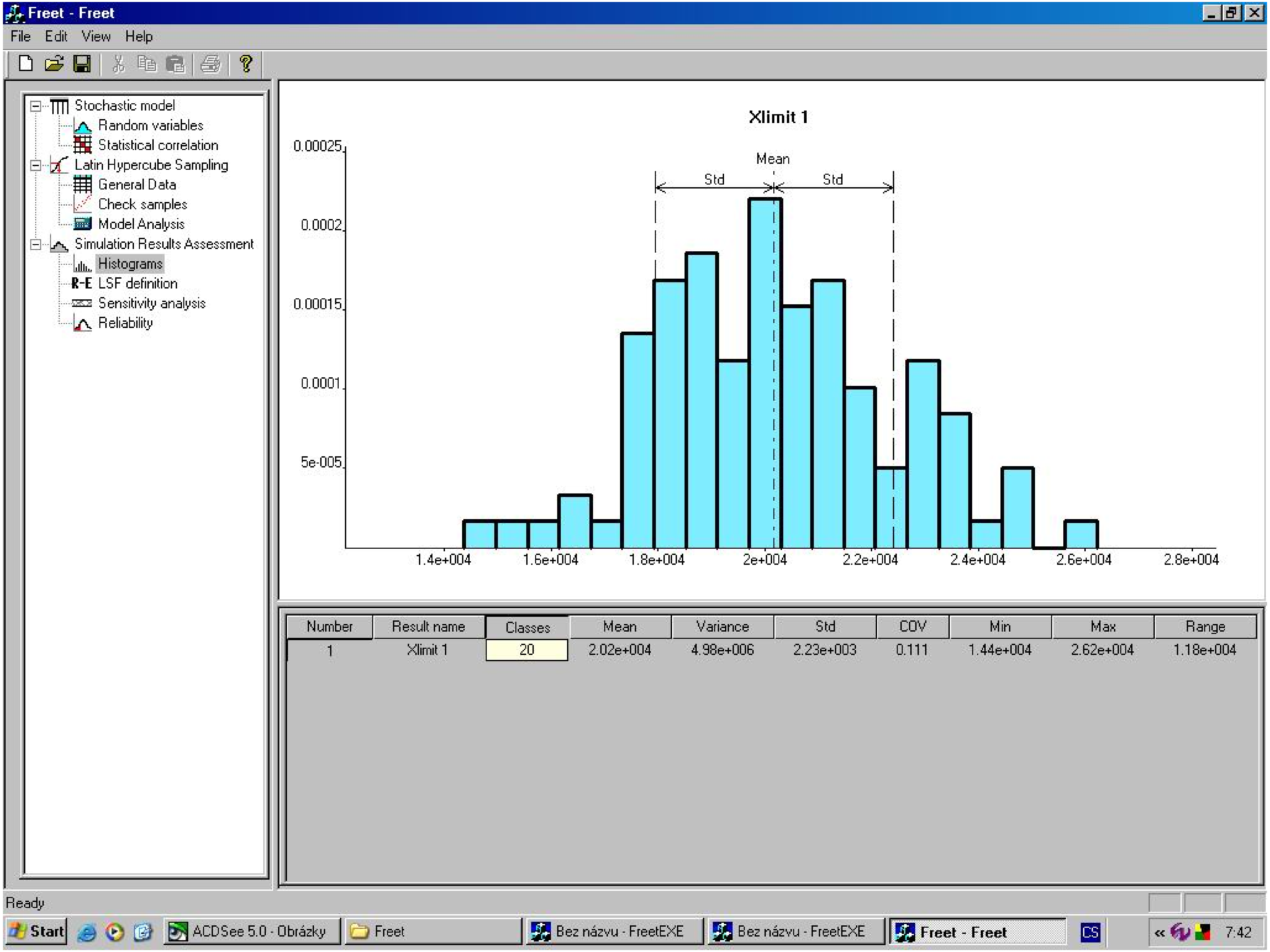
Compressive strength vs. Tensile Strength

Correlation:
Requested: 0.8
Obtained: 0.799
Error: 0.000766

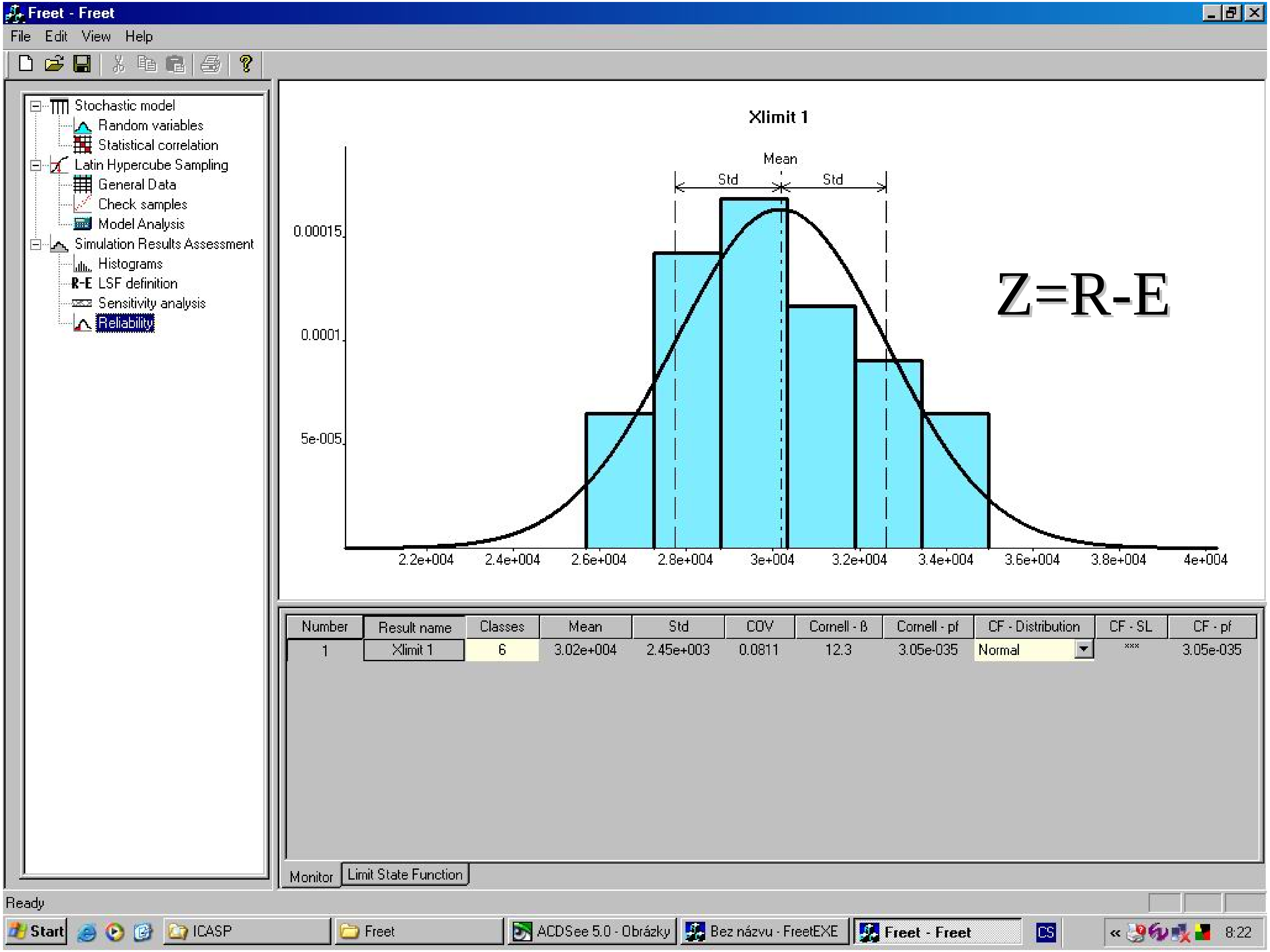
	Calibration factor cip	Compressive strength	On site concrete strength	Placing, curing, hardening	Tensile Strength	Embedmen
Calibration factor cip	1	0	0	0	0	0
Compressive strength	0	1	0	0	0.8	0
On site concrete strength	0	-0.00196	1	0	0	0
Placing, curing, hardening	0	-0.00122	-0.00111	1	0	0
Tensile Strength	0	0.799	0.00437	-0.000797	1	0
Embedment Depth	0	-0.000593	-0.00129	0.000945	-0.00129	1
Model uncertainty	0	-0.00247	0.000834	-0.00346	-0.000536	0.0002
Crack Width	0	-0.000916	0.00064	0.00094	0.00185	-0.0005
Fastener Diameter	0	0	0	0	0	0



Number	Name of the DLL	Exported functions	Result name
1	XlimitF.dll	XLIMITF	Xlimit 1



Number	Result name	Classes	Mean	Variance	Std	COV	Min	Max	Range
1	Xlimit 1	20	2.02e+004	4.98e+006	2.23e+003	0.111	1.44e+004	2.62e+004	1.18e+004



Z=R-E

Number	Result name	Classes	Mean	Std	COV	Cornell - ß	Cornell - pf	CF - Distribution	CF - SL	CF - pf
1	Xlimit 1	6	3.02e+004	2.45e+003	0.0811	12.3	3.05e-035	Normal	***	3.05e-035

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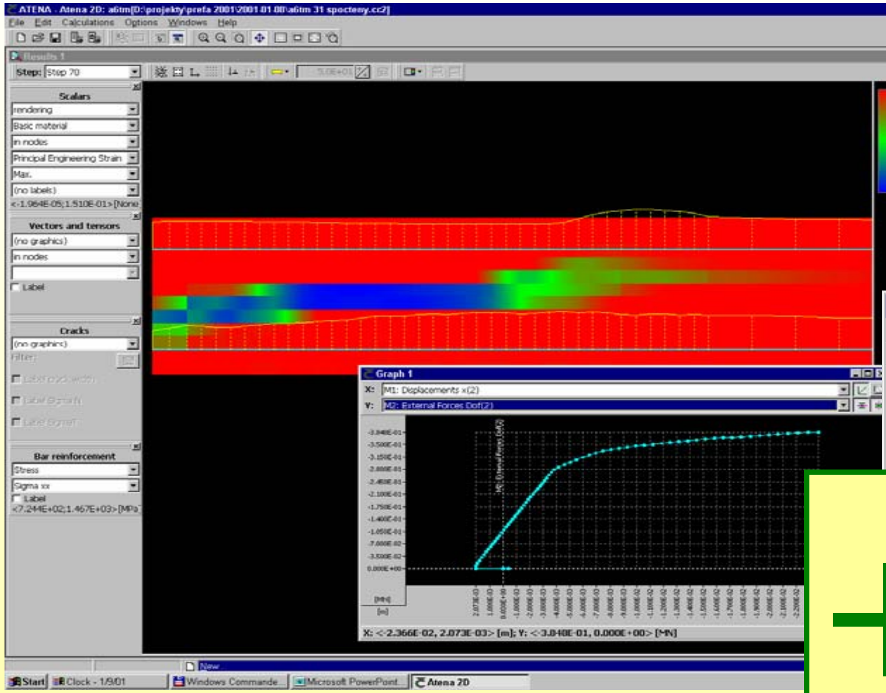
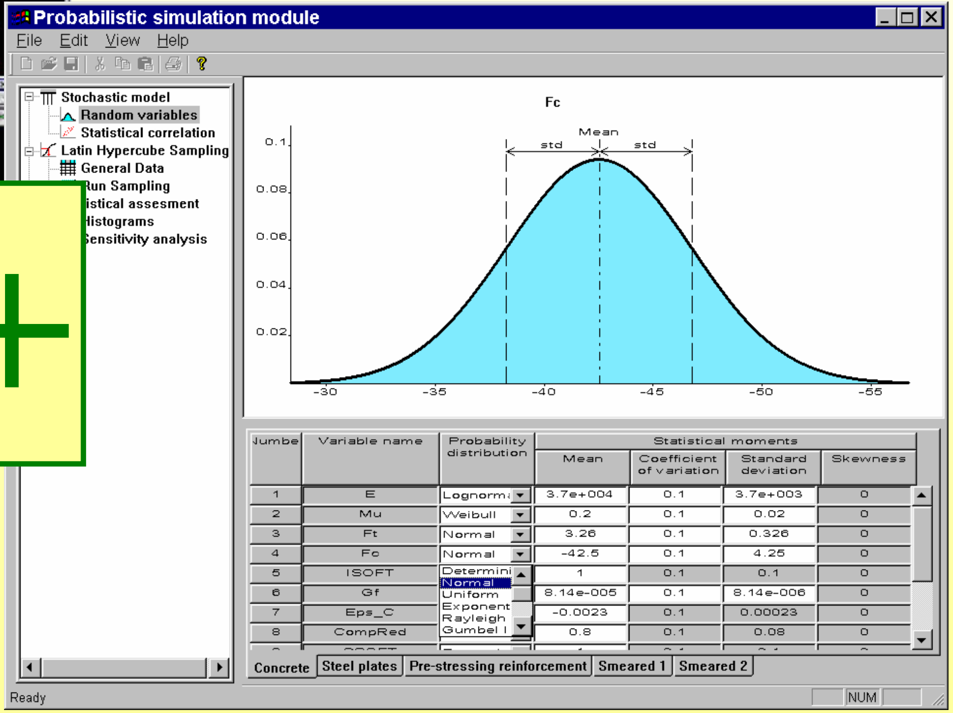
Stochastic NLFEM – SARA Studio



Probabilistic software FReeT



Jumbe	Variable name	Probability distribution	Statistical moments			
			Mean	Coefficient of variation	Standard deviation	Skewness
1	E	Lognorm	3.7e+004	0.1	3.7e+003	0
2	Mu	Weibull	0.2	0.1	0.02	0
3	Ft	Normal	3.26	0.1	0.326	0
4	Fc	Normal	-42.5	0.1	4.25	0
5	ISOFT	Determini Normal Uniform Exponent Rayleigh Gumbel I	1	0.1	0.1	0
6	Gf		8.14e-005	0.1	8.14e-006	0
7	Eps_C		-0.0023	0.1	0.00023	0
8	CompRed		0.8	0.1	0.08	0

\+

Software for nonlinear fracture mechanics analysis ATENA

Non-linear techniques and material models for concrete: ATENA software

Numerical core – advanced nonlinear material models

concrete in tension

tensile cracks

post-peak behavior

Crack band size: L

$$\varepsilon = \frac{w}{L}$$

smeared crack approach

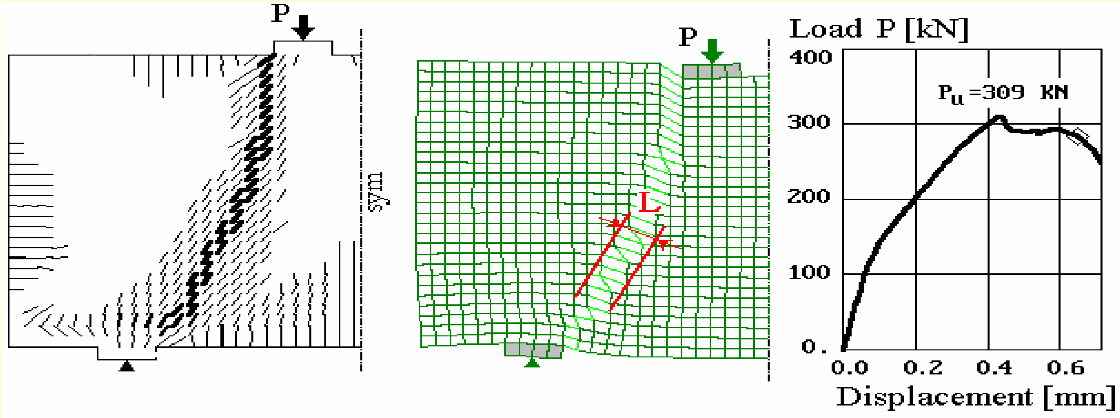
crack band method

fracture energy

fixed or rotated cracks

crack localization

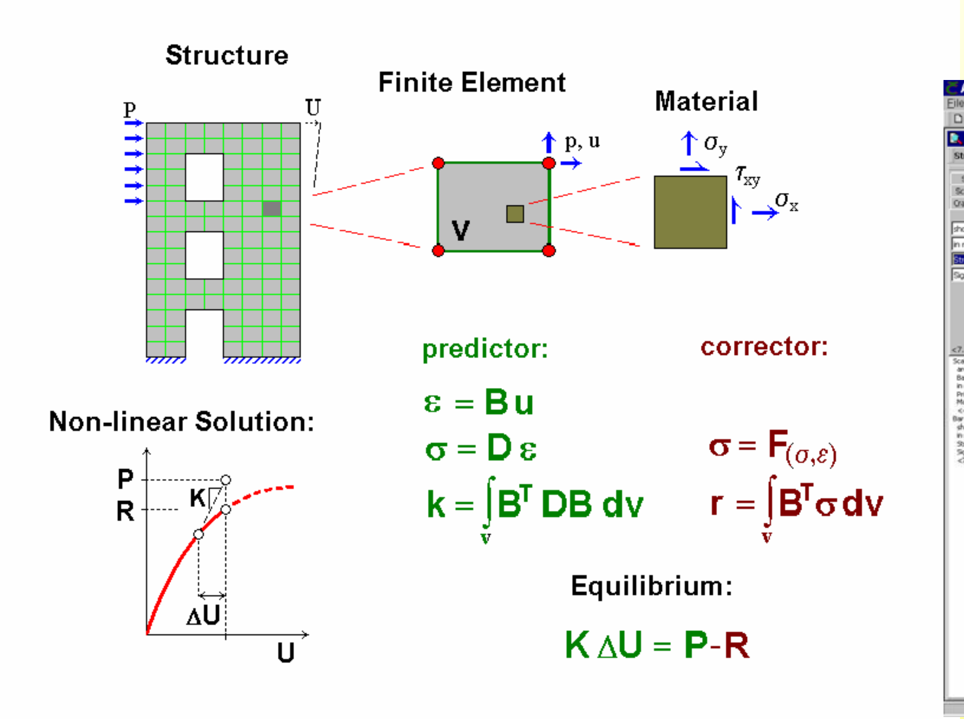
size-effect is captured



Software ATENA

Well-balanced approach for practical applications of advanced FEM in civil engineering

Numerical core – state-of-art background + user friendly Graphical user environment – visualization + interaction



predictor:

$$\varepsilon = B u$$

$$\sigma = D \varepsilon$$

$$k = \int_V B^T D B \, dv$$

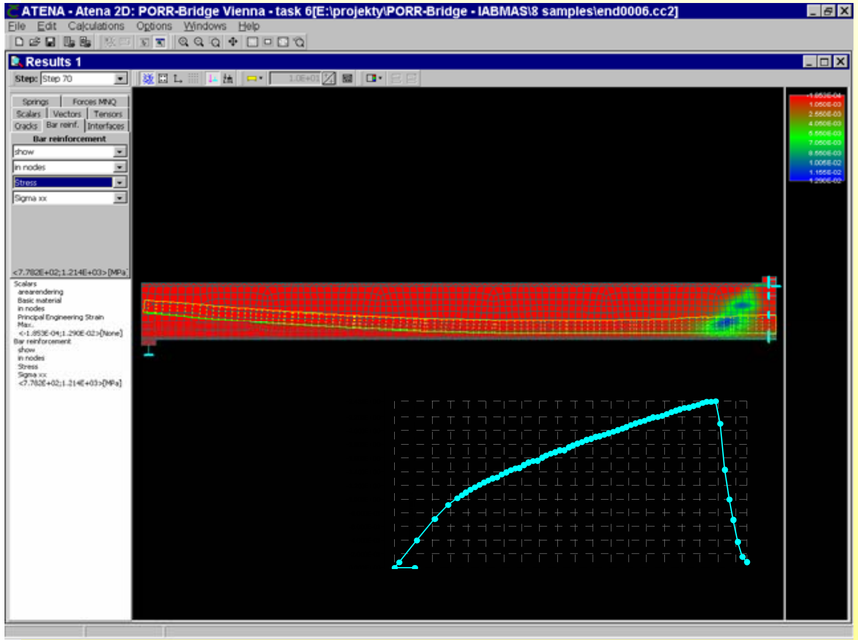
corrector:

$$\sigma = F_{(\sigma,\varepsilon)}$$

$$r = \int_V B^T \sigma \, dv$$

Equilibrium:

$$K \Delta U = P - R$$



Crack band method

Numerical core – advanced nonlinear material models

concrete in tension

tensile cracks

post-peak behavior

smeared crack approach

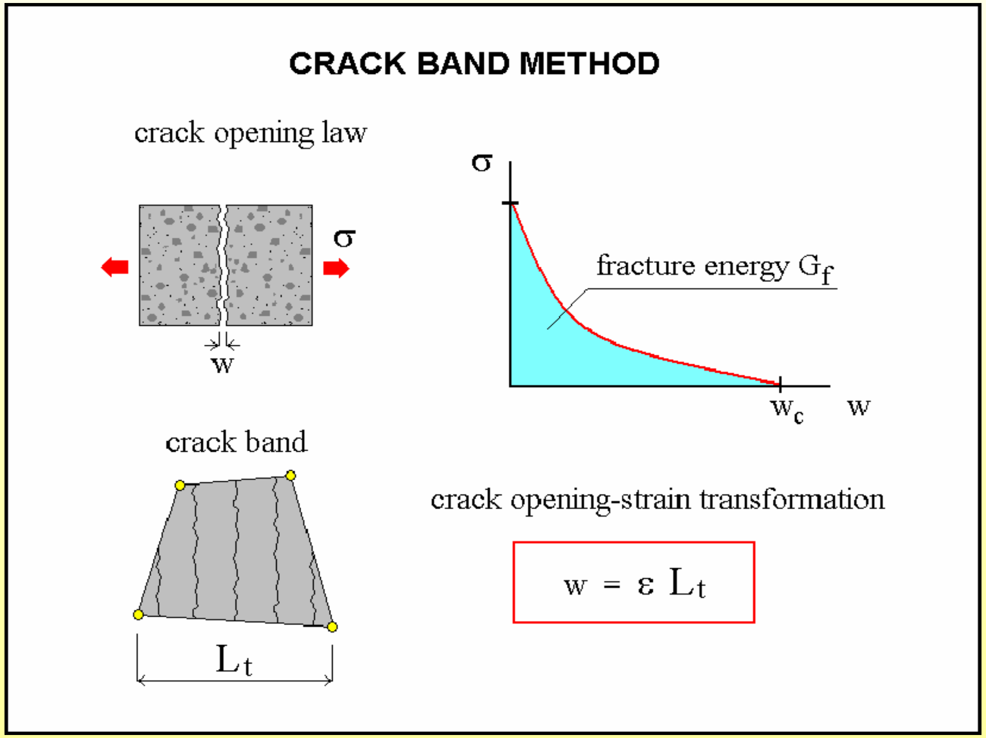
crack band method

fracture energy

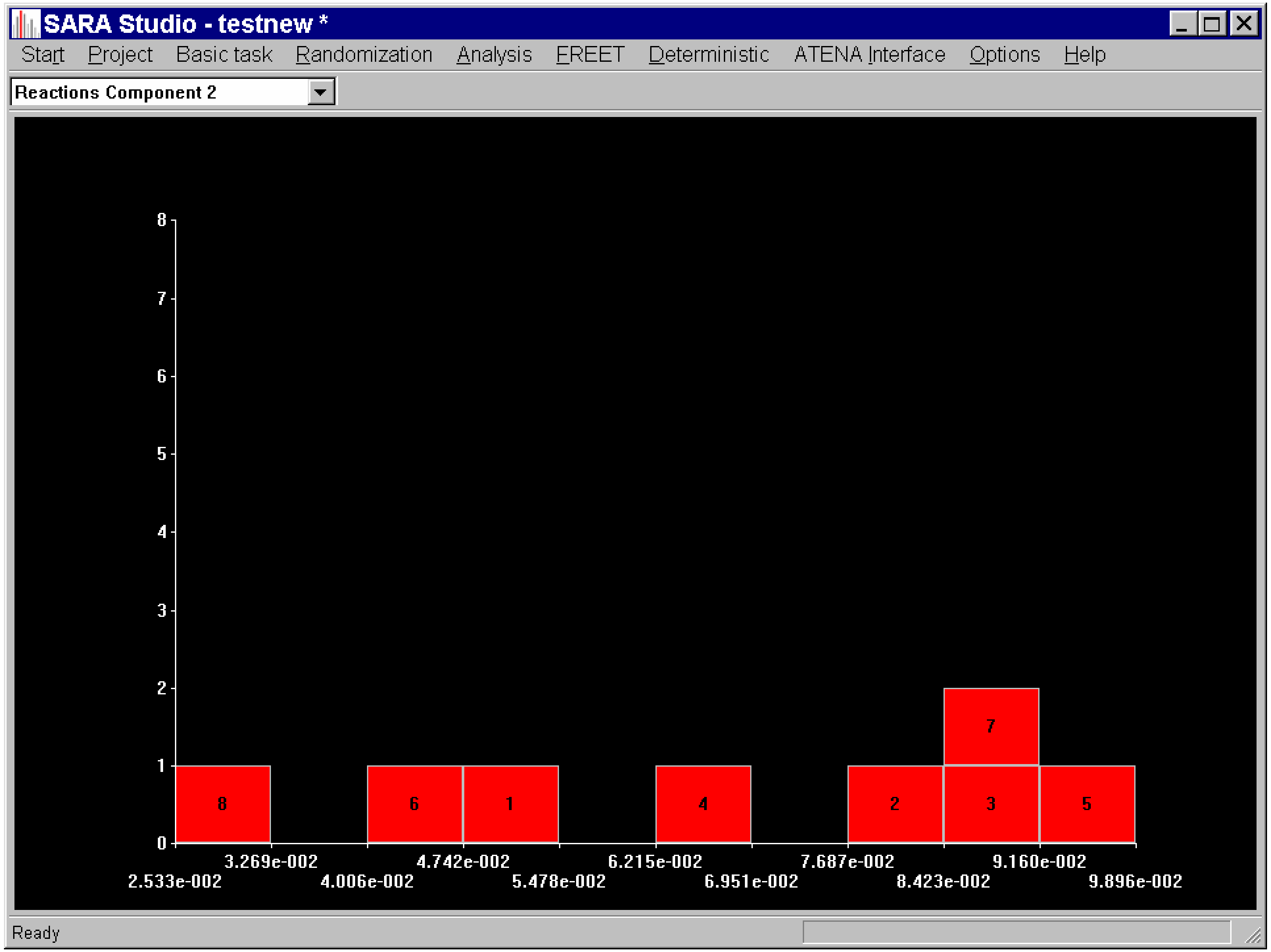
fixed or rotated cracks

crack localization

size-effect is captured

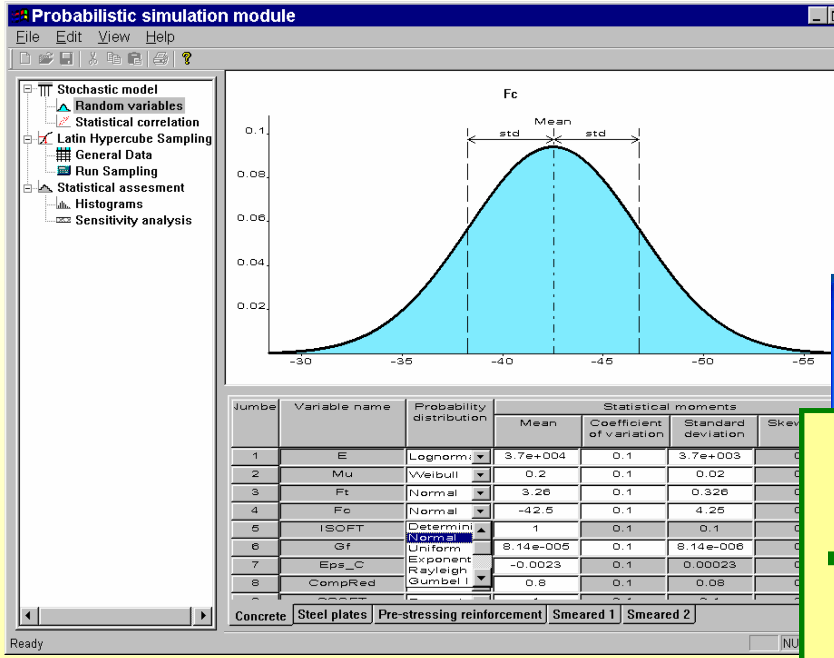


- Run-time
- histogram
- of results



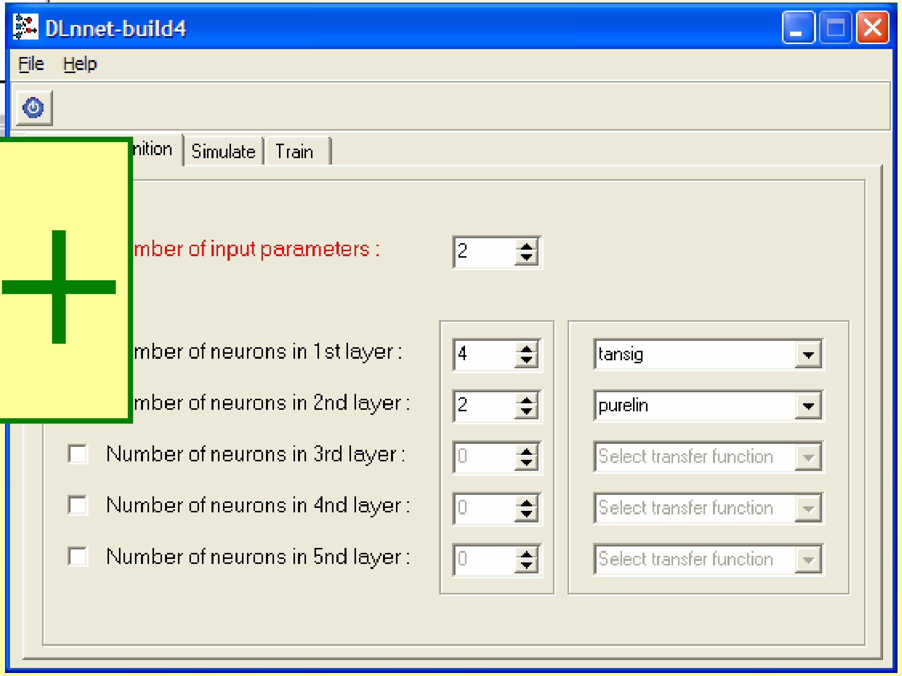
13.4.2007

Software tools



Jumbe	Variable name	Probability distribution	Statistical moments		
			Mean	Coefficient of variation	Standard deviation
1	E	Lognorm	3.7e+004	0.1	3.7e+003
2	Mu	Weibull	0.2	0.1	0.02
3	Ft	Normal	3.26	0.1	0.326
4	Fc	Normal	-42.5	0.1	4.25
5	ISOFT	Determini Normal Uniform Exponent Rayleigh Gumbel I	1	0.1	0.1
6	Gf		8.14e-005	0.1	8.14e-006
7	Eps_C		-0.0023	0.1	0.00023
8	CompRed		0.8	0.1	0.08

Software DLNNET: neural networks



mber of input parameters : 2

mber of neurons in 1st layer : 4 tansig

mber of neurons in 2nd layer : 2 purelin

☐ Number of neurons in 3rd layer : 0 Select transfer function

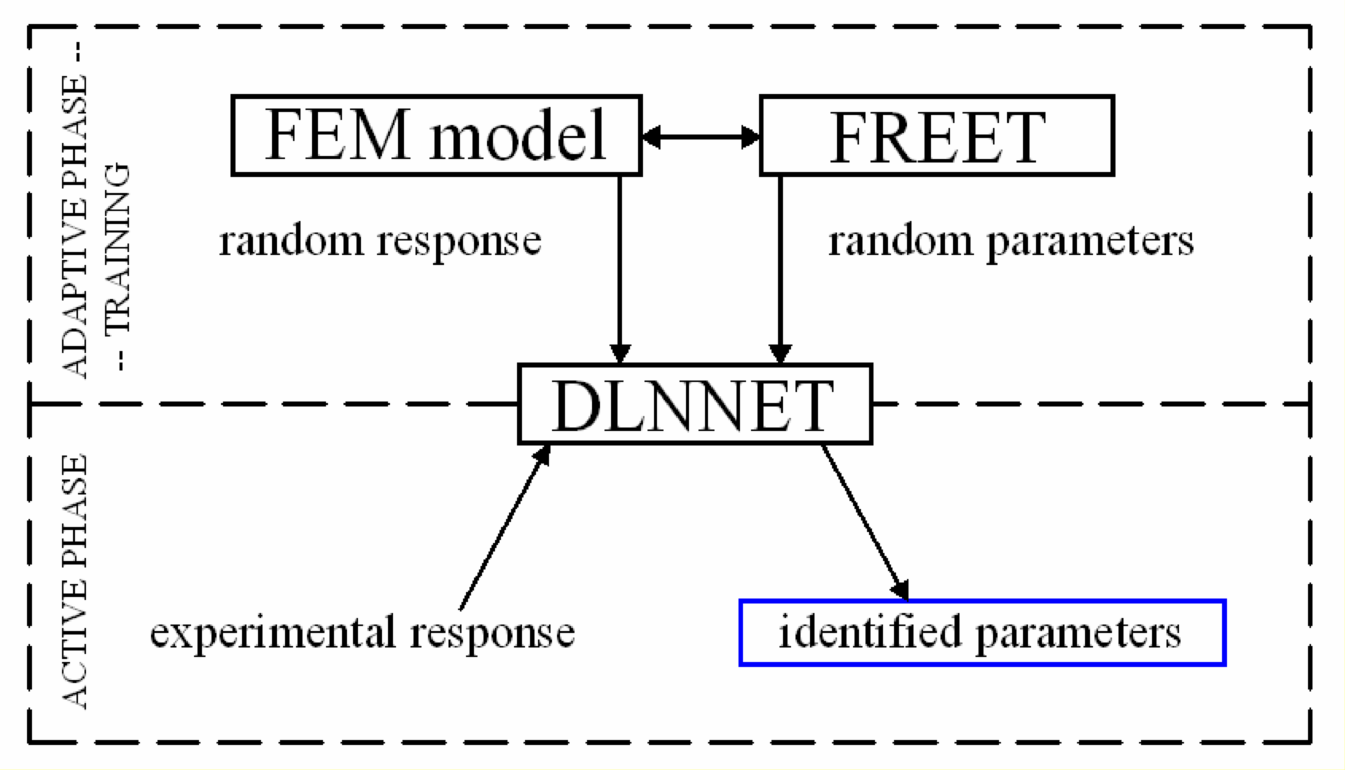
☐ Number of neurons in 4nd layer : 0 Select transfer function

☐ Number of neurons in 5nd layer : 0 Select transfer function

Software FReET: statistical, sensitivity and reliability analyses

http://www.freet.cz

Software communication for inverse analysis



ADAPTIVE PHASE -- TRAINING --

FEM model

FREET

random response

random parameters

DLNNET

ACTIVE PHASE

experimental response

identified parameters

Selected types of applications

Example of FReET stand-alone application:

- Statistical analysis of concrete subway tunnel under Vltava river

SARA - classes of tasks:

- Probabilistic analyses of concrete structures
- Statistical size effect studies
- Verification of (code) design formulas
- Identification of material model parameters (inverse analysis)

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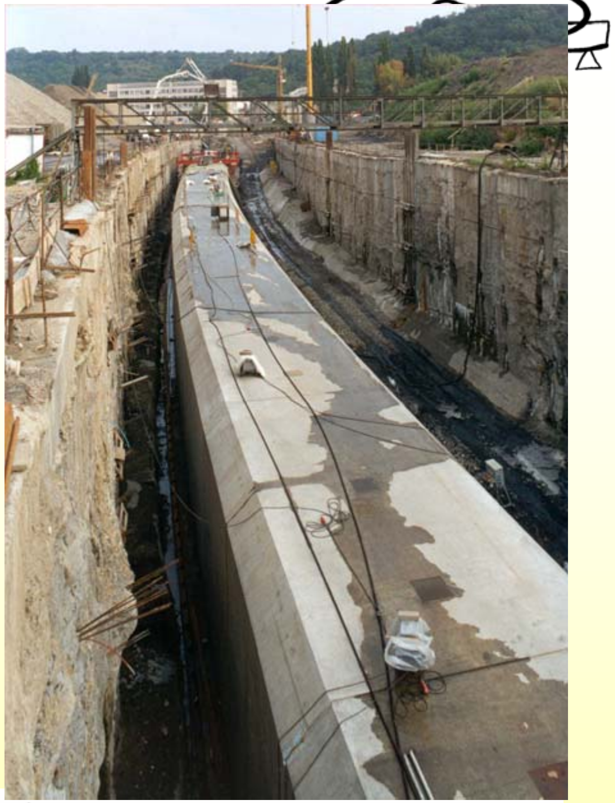
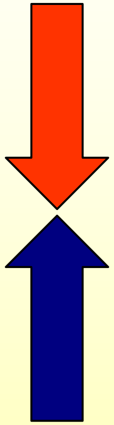
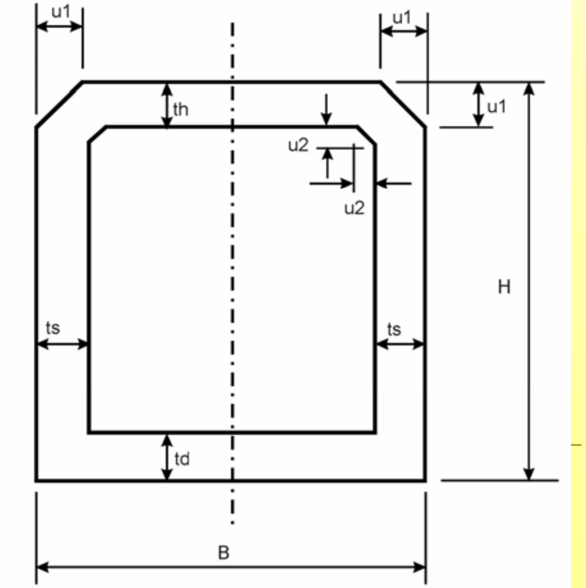
Large concrete subway tunnel under Vltava river in Prague (2002)



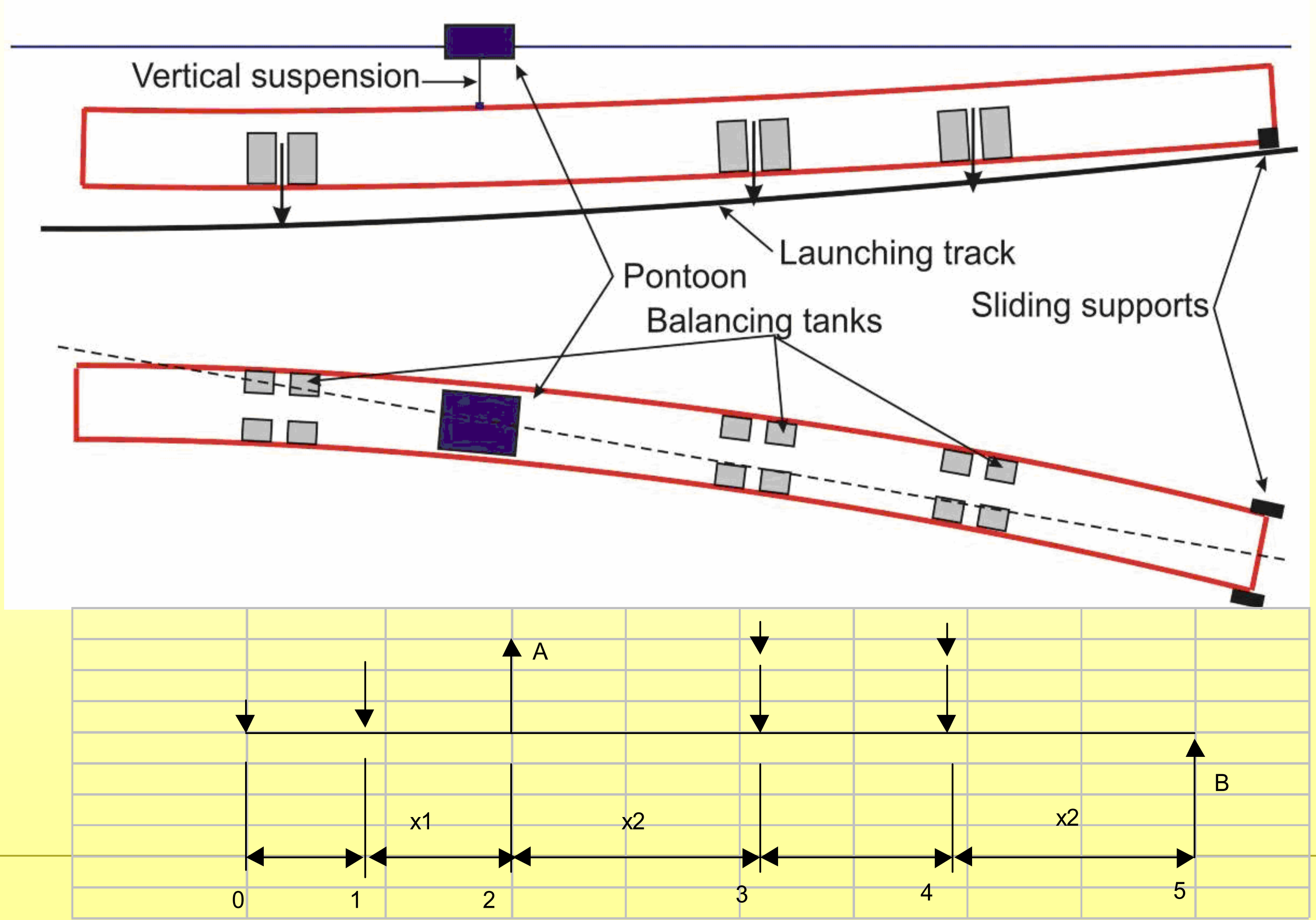
- Weight of tunnel
- Uplift force



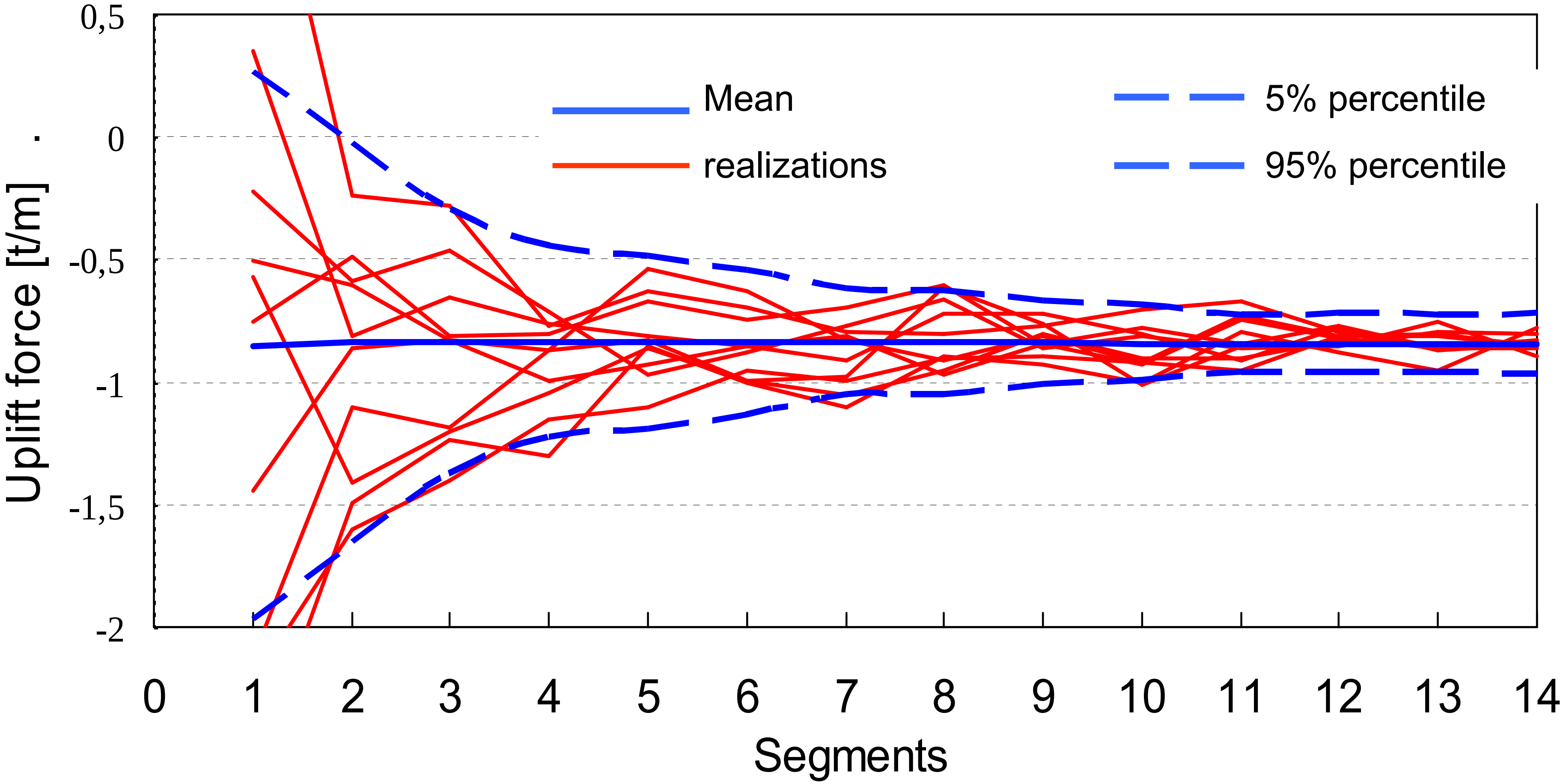
- 211 random variables
- Imperfection of geometry, 14 segments
- Target: risk minimization
- Updating segments - convergence to required uplift force



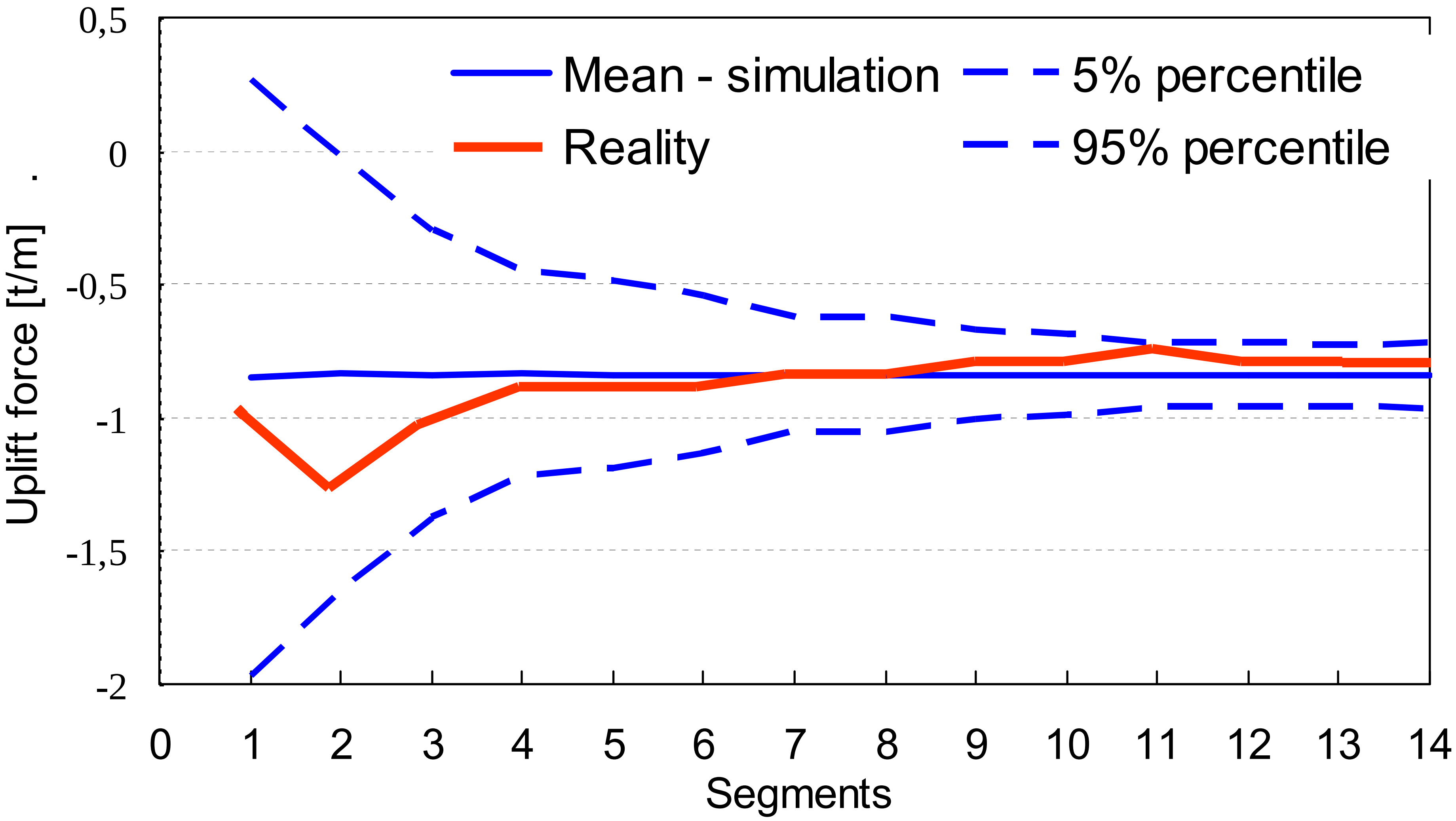
Static scheme, forces acting on the tube



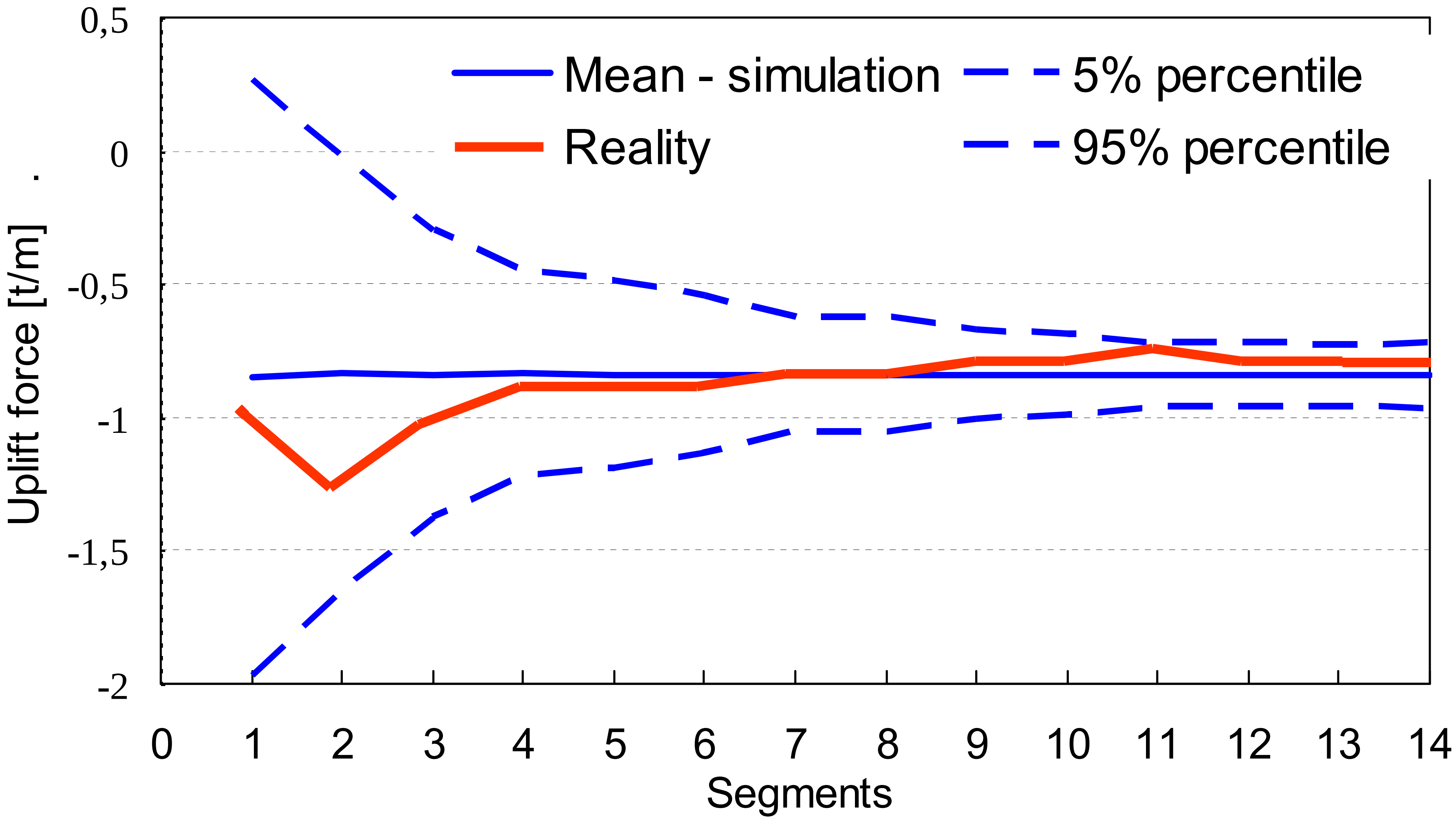
Statistical simulation of uplift force



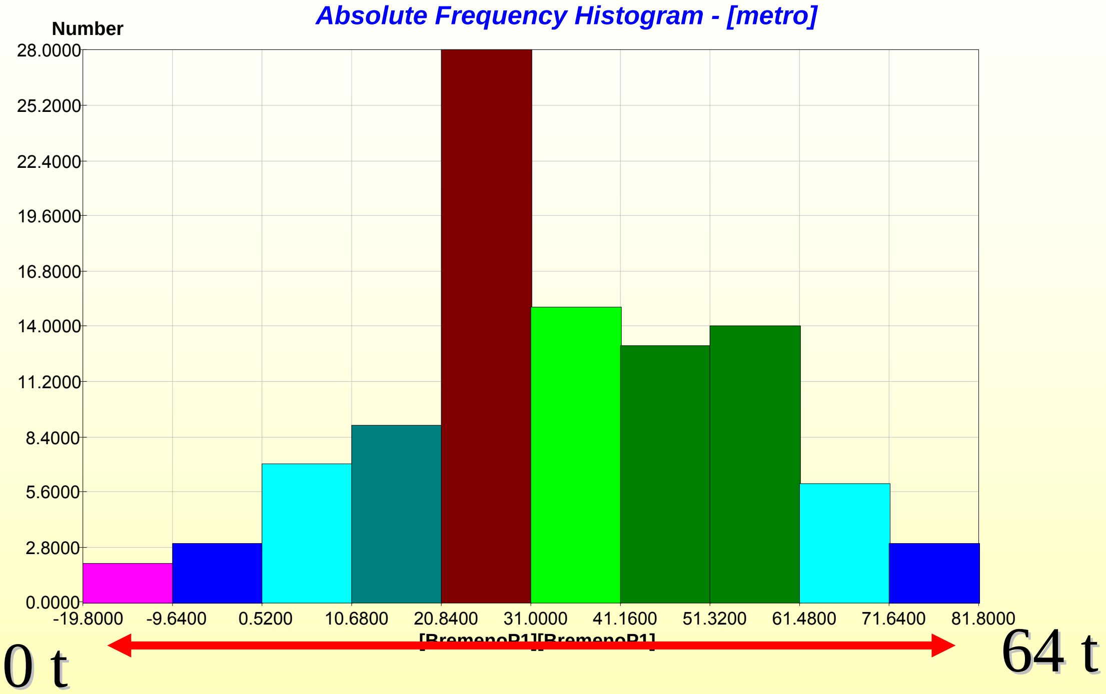
Statistical simulation and measurement

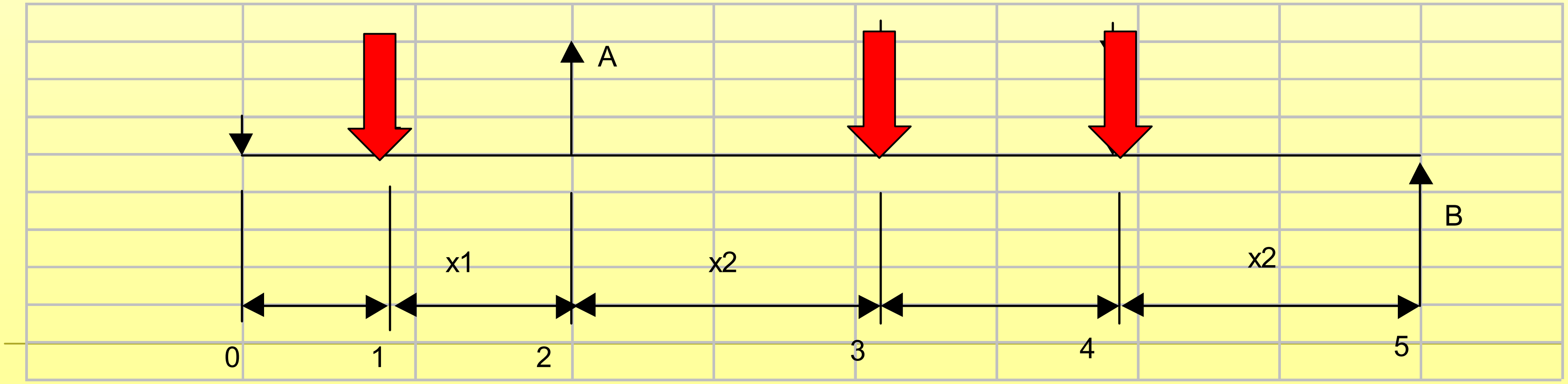


Statistical simulation and measurement



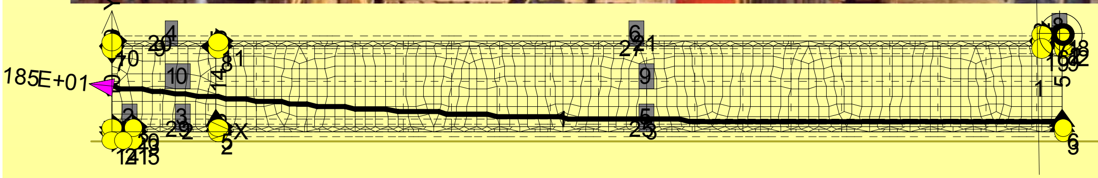
Forces - barrels with water

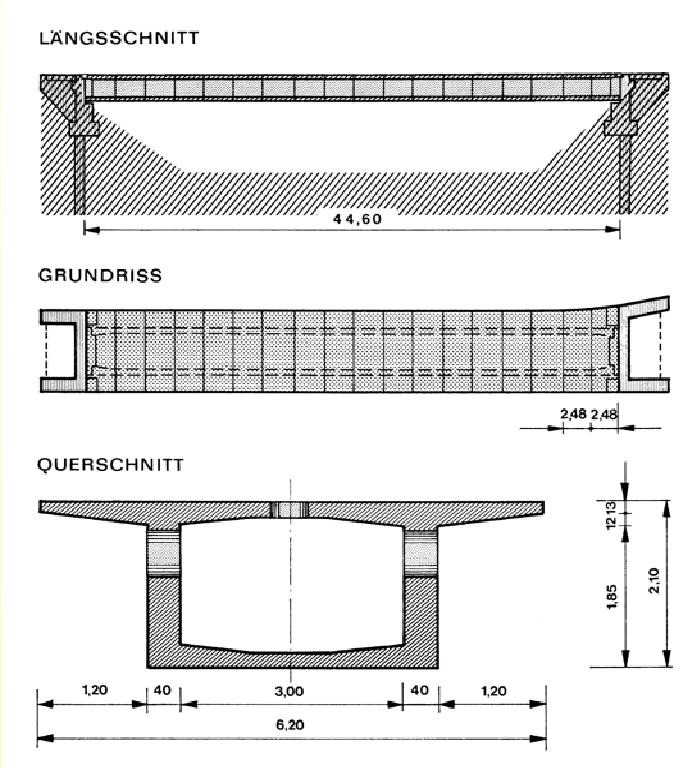




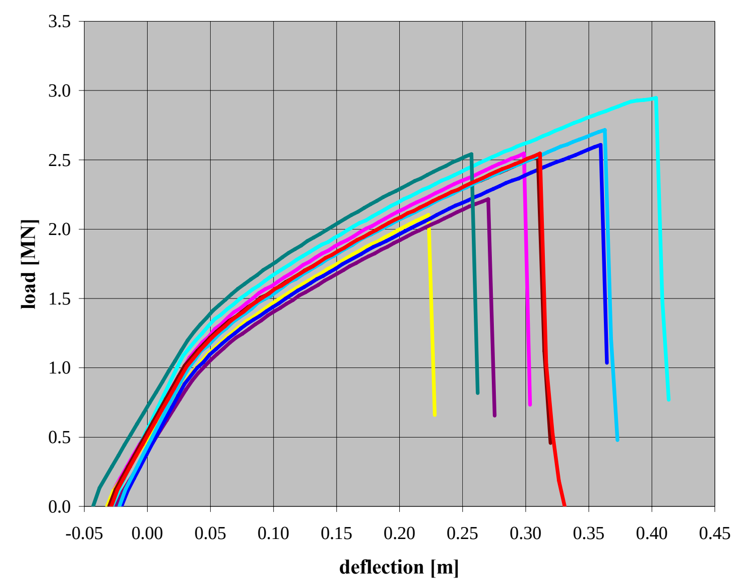
Probabilistic analyses of concrete structures:

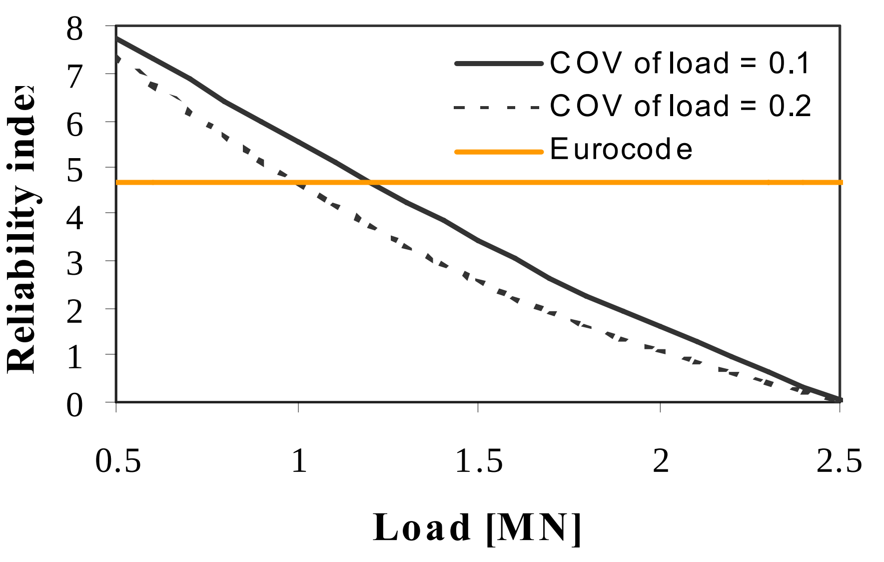
Box-girder prestressed bridge in Vienna





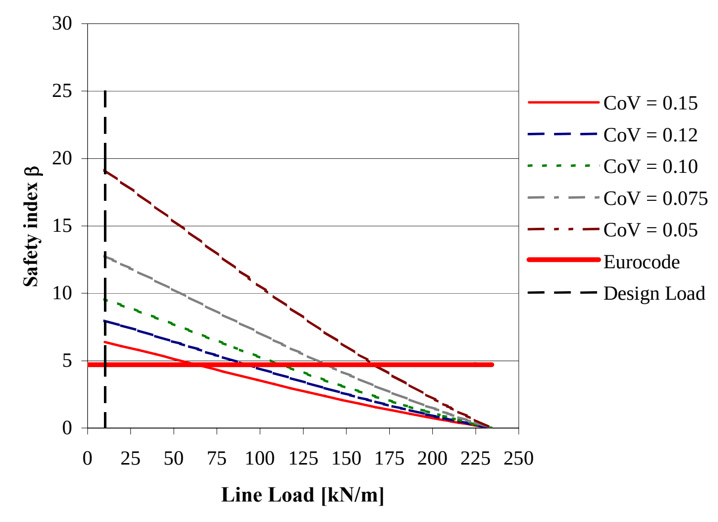
Random variable description	Symbol	Units	Mean value	COV	Distribution type	Reference
Concrete grade B500						
Modulus of elasticity	E_c	GPa	36.95	0.15	Lognormal	[6]
Poisson's ratio	μ	-	0.2	0.05	Lognormal	Estimation
Tensile strength	f_t	MPa	3.257	0.18	Weibull	[6]
Compressive strength	f_c	MPa	42.5	0.10	Lognormal	[6,7]
Specific fracture energy	G_f	N/m	81.43	0.20	Weibull	[8]
Uniaxial compressive strain	ε_c	-	0.0023	0.15	Lognormal	[6]
Reduction of strength	c_{Red}	-	0.8	0.06	Rectangular	Estimation
Critical comp displacement	w_d	m	0.0005	0.10	Lognormal	Estimation
Specific material weight	ρ	MN/m^3	0.023	0.10	Normal	[9]
Prestressing strands						
Modulus of elasticity	E_s	GPa	200.0	0.03	Lognormal	[10]
Yield stress	f_y	MPa	1600.0	0.07	Lognormal	[10]
Prestressing force	F	MN	21.85	0.04	Normal	[9]
Area of strands	A_s	m^2	0.0237	0.001	Normal	[9]





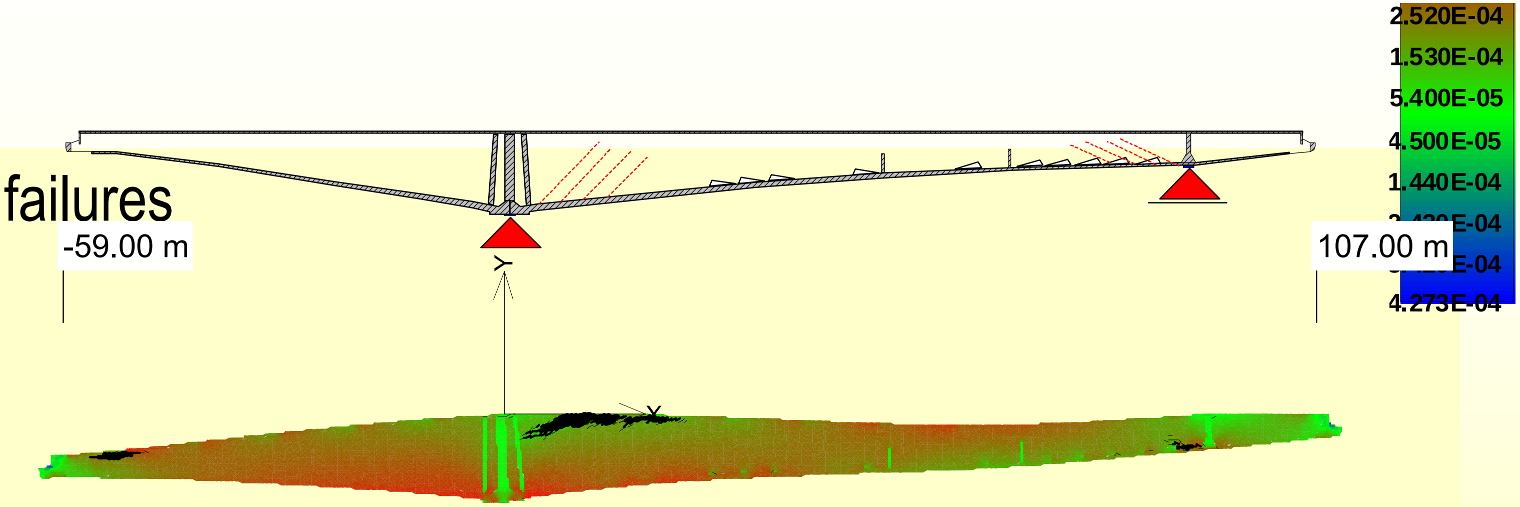
Probabilistic analyses of concrete structures:
Cantilever beam bridge in Italy

Colle d’Isarco bridge. Brennero highway, Italy

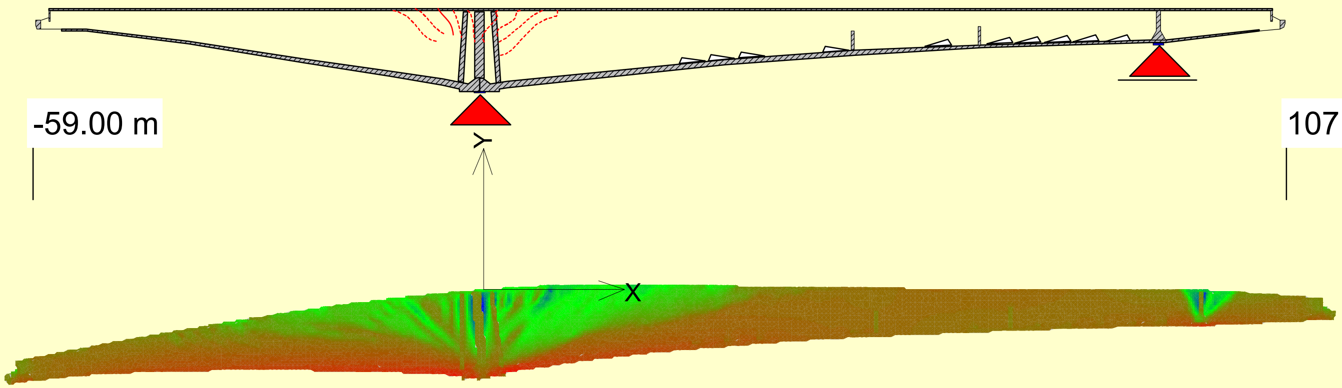


Reliability index vs. load

- Typical failures



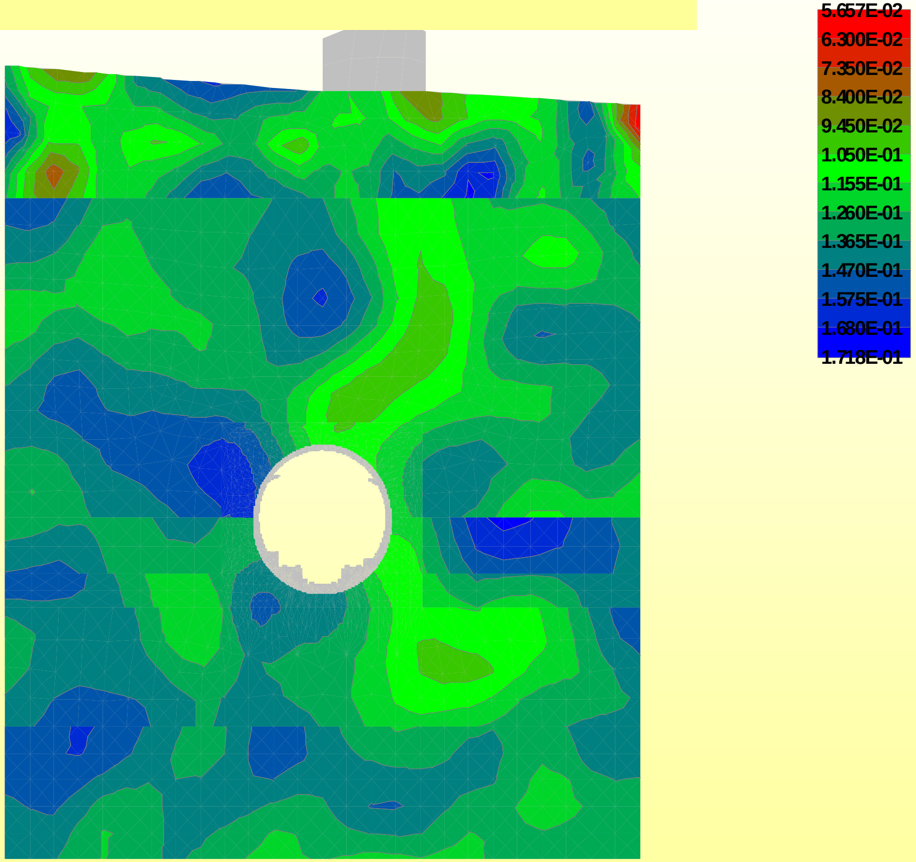
- Shear



Probabilistic analyses of concrete structures:

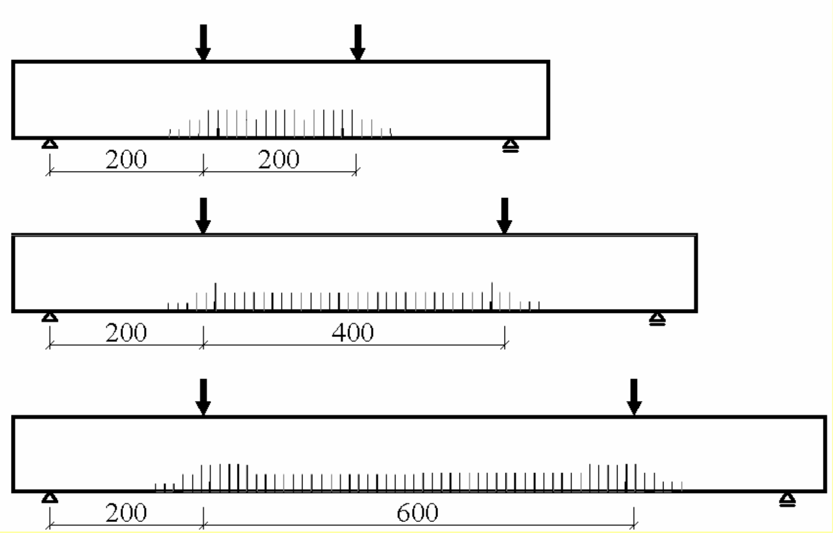
Soil-structure interaction, spatial variability

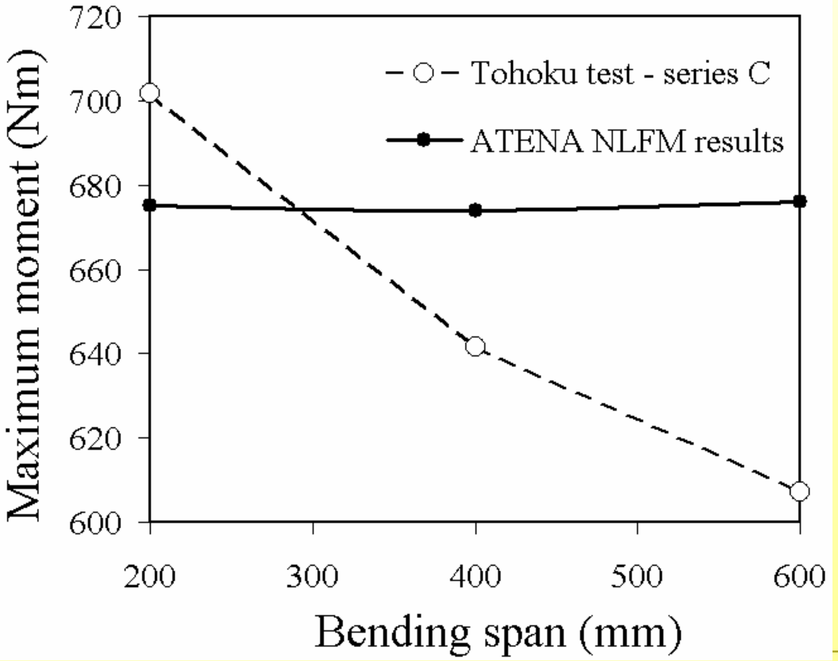
- Stability of concrete tunnel tube in complicated geological conditions
- Influence of spatial variability of Young modulus and material constants of Drucker-Prager criterion (based on cohesion and angle of internal friction)
- Analyzed part 50 x 60m, diametr of tunnel 11m, wall thickness 0.5m
- Plain strain state, 5000 finite elements

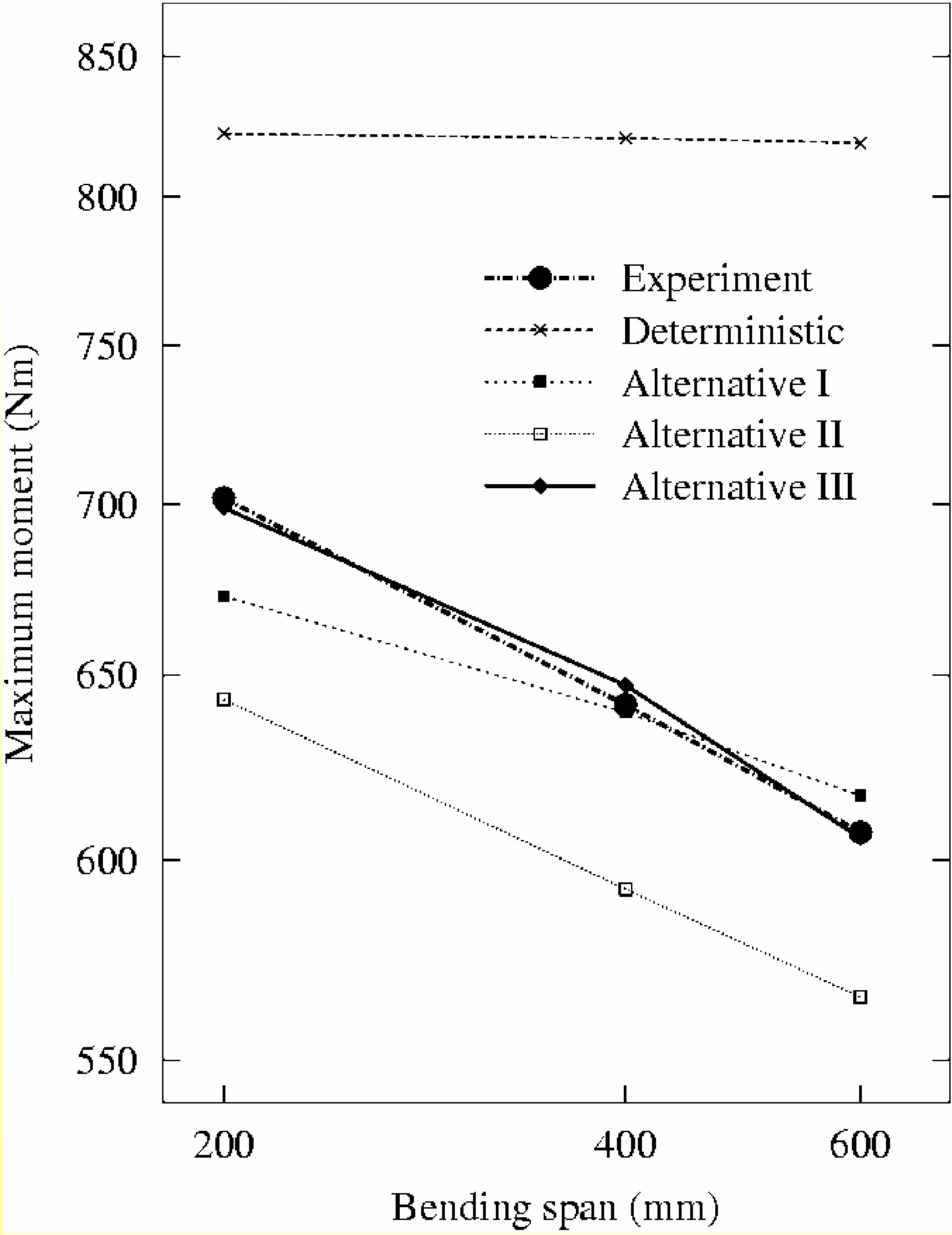


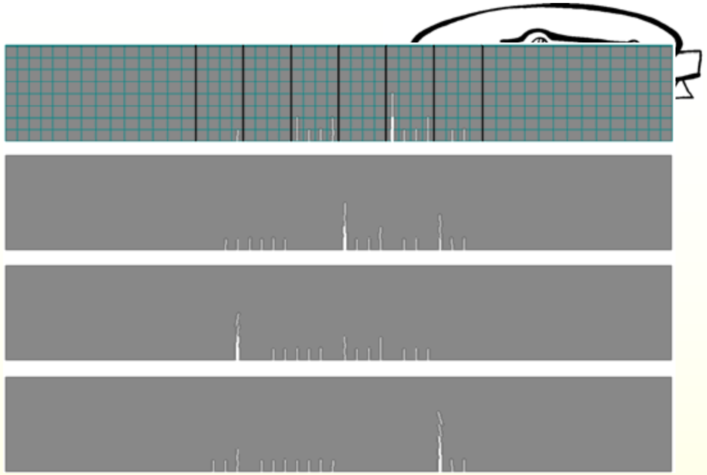
Statistical size effect studies: Four-point bending - different bending span

- Koide at al. Experiments on 4PB
- Statistical size effect!
- Cannot be captured at deterministic level

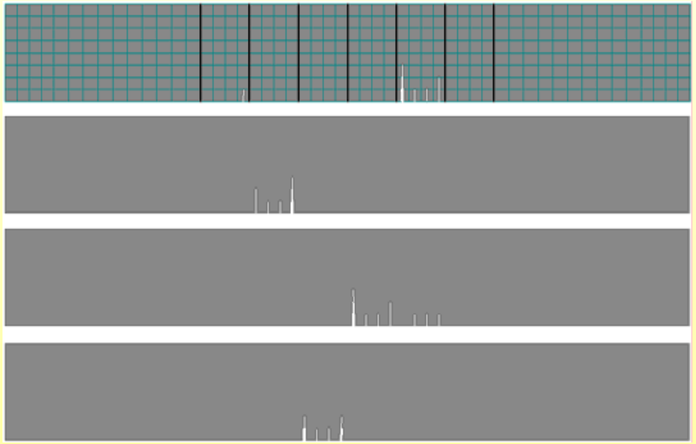






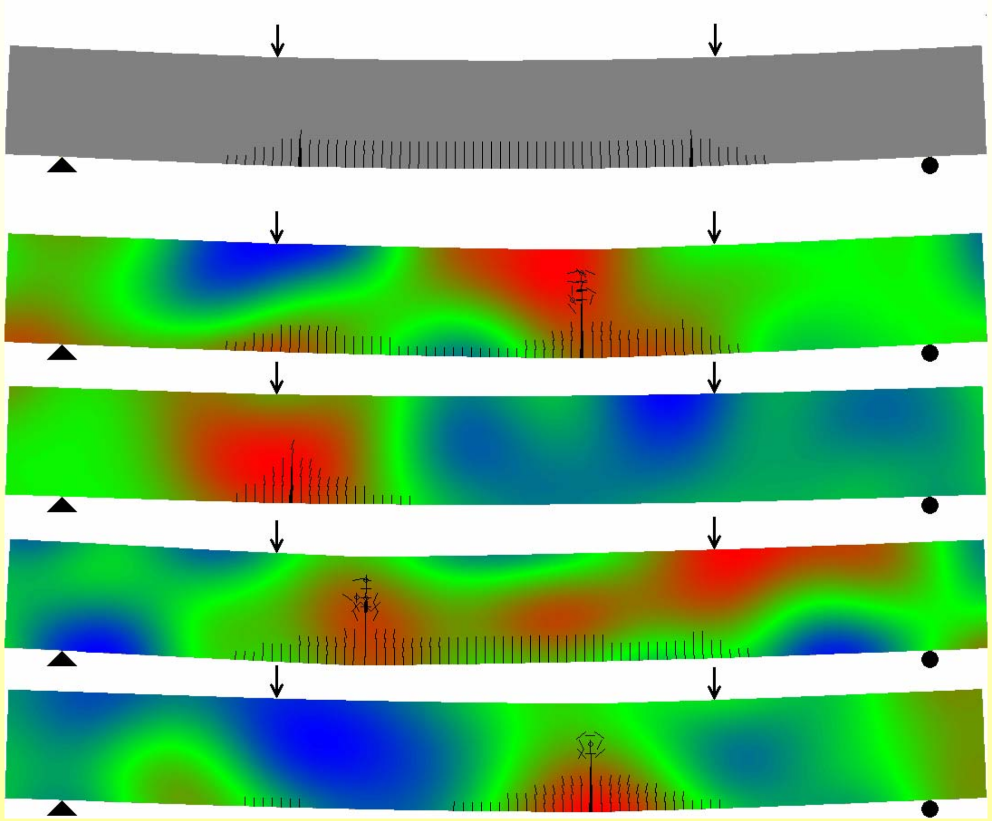


Alt. I: No correlation between tensile strength and fracture energy

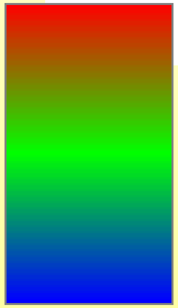


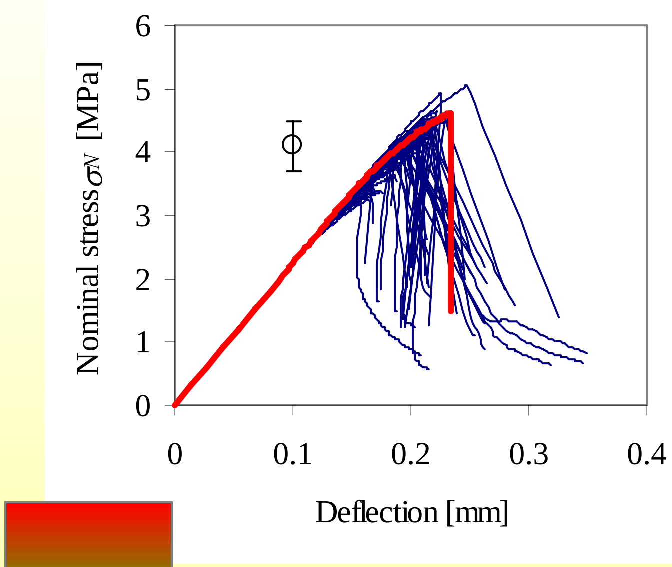
Alt. II, III: High correlation between tensile strength and fracture energy

Statistical size effect studies: Four-point bending - different bending span



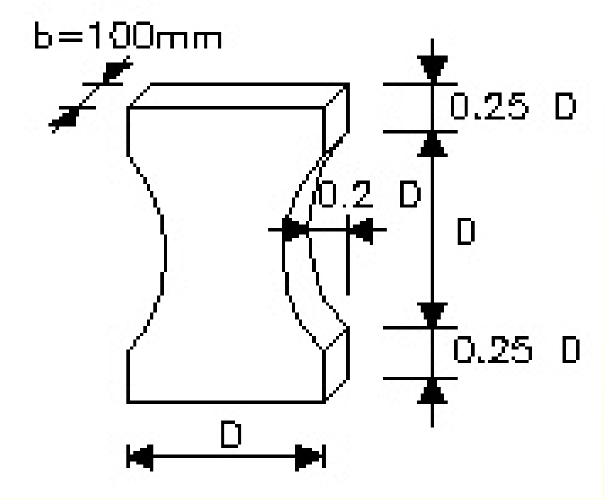
Four-point bending and patterns of random fields





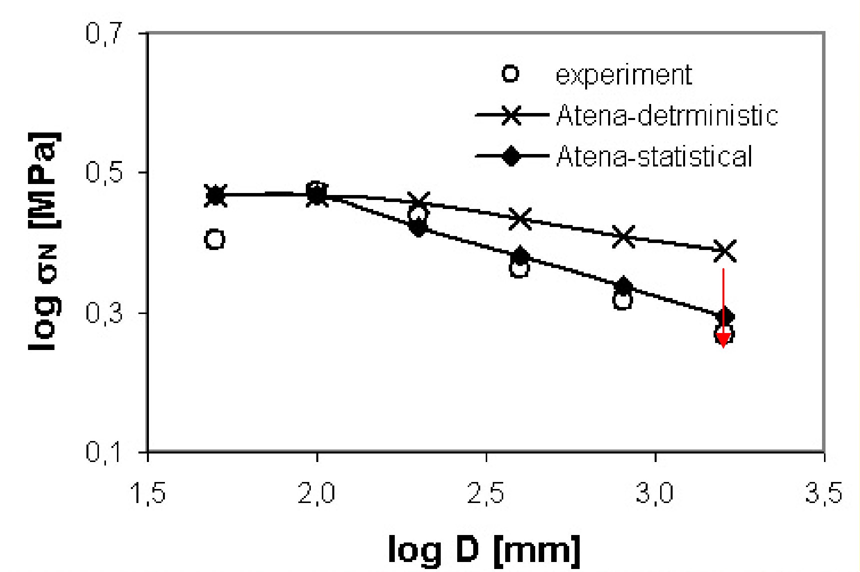
Random load – deflection curves (red curve – deterministic calculation).

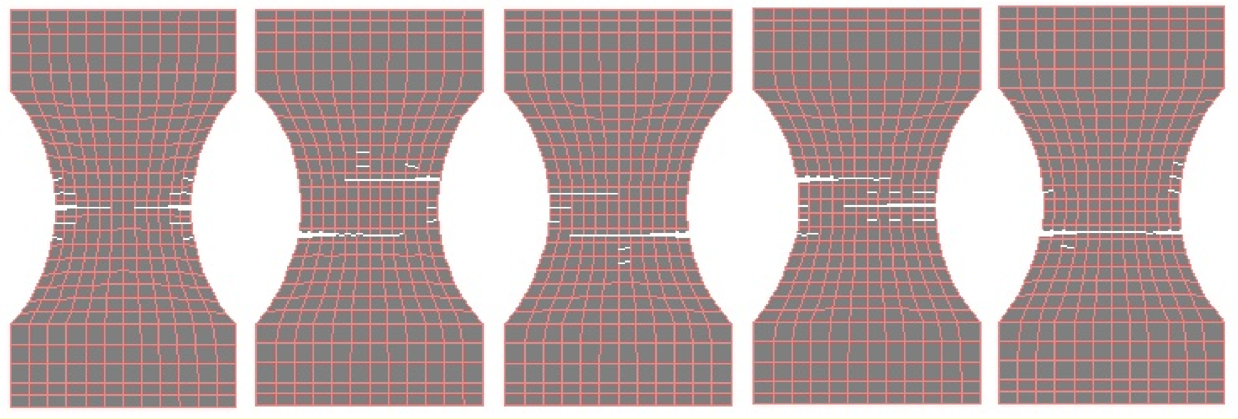
Statistical size effect studies: Dog-bone shaped concrete specimens in uniaxial tension



Size	D [mm]
A	50
B	100
C	200
D	400
E	800
F	1600

EXPERIMENT - Van Vliet and Van Mier, 1997

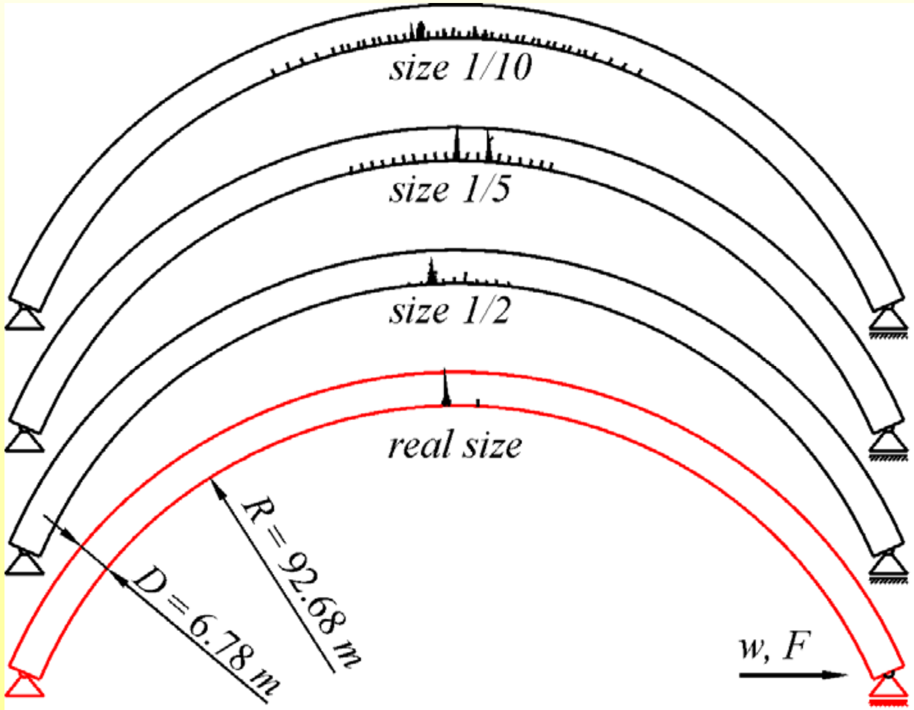


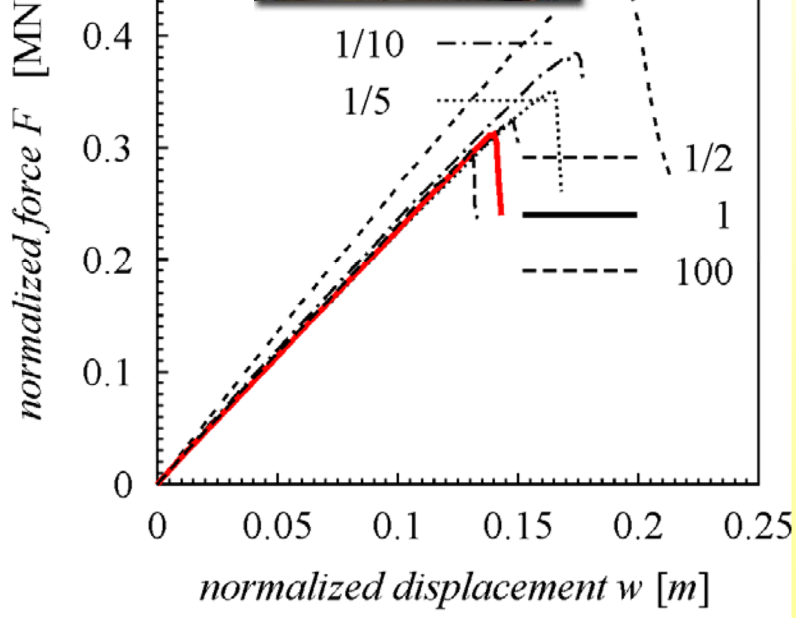


Statistical size effect studies: Malpasset dam (failed 1959)

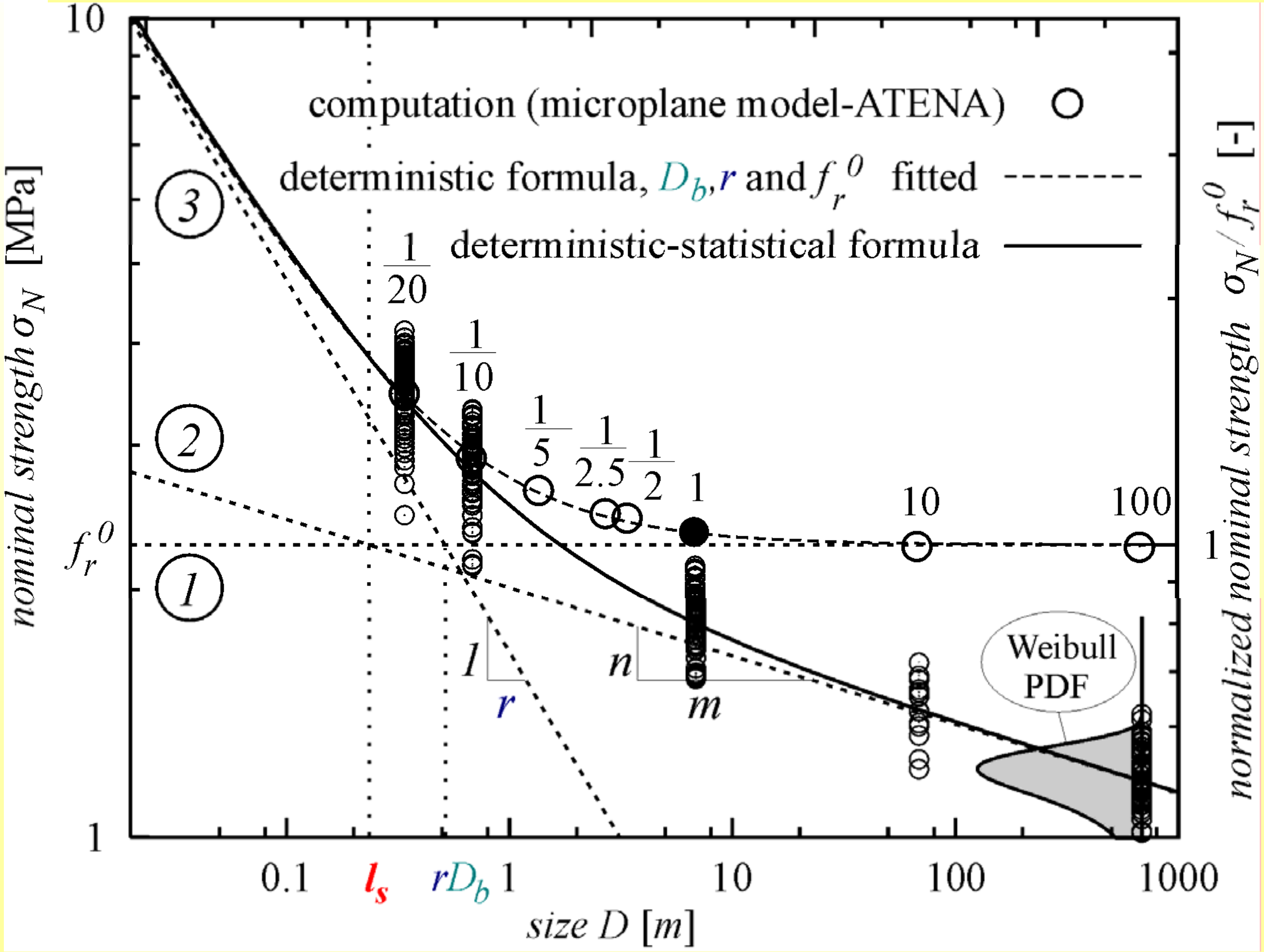
Calculation for different sizes, microplane M4 model

Bažant, Vořechovský, Novák - Icossar 2005

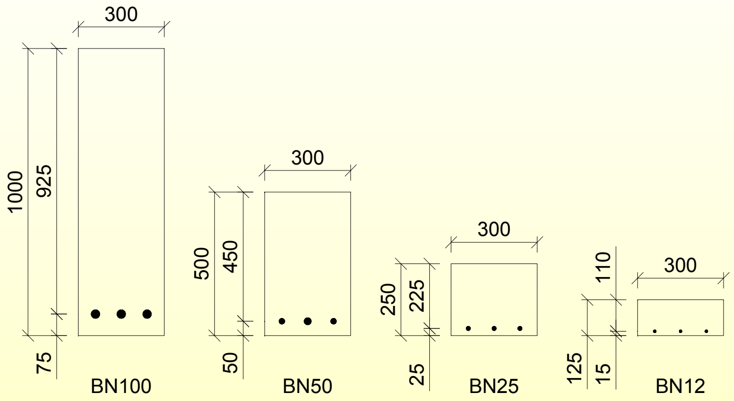


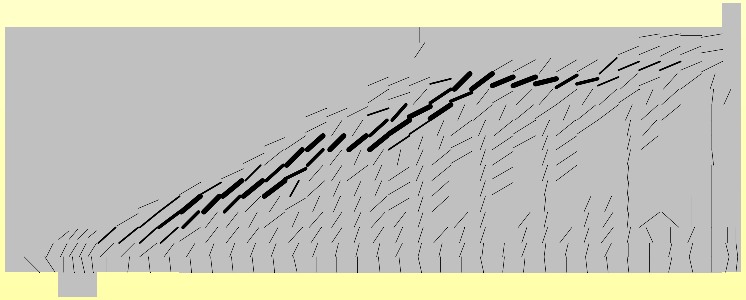


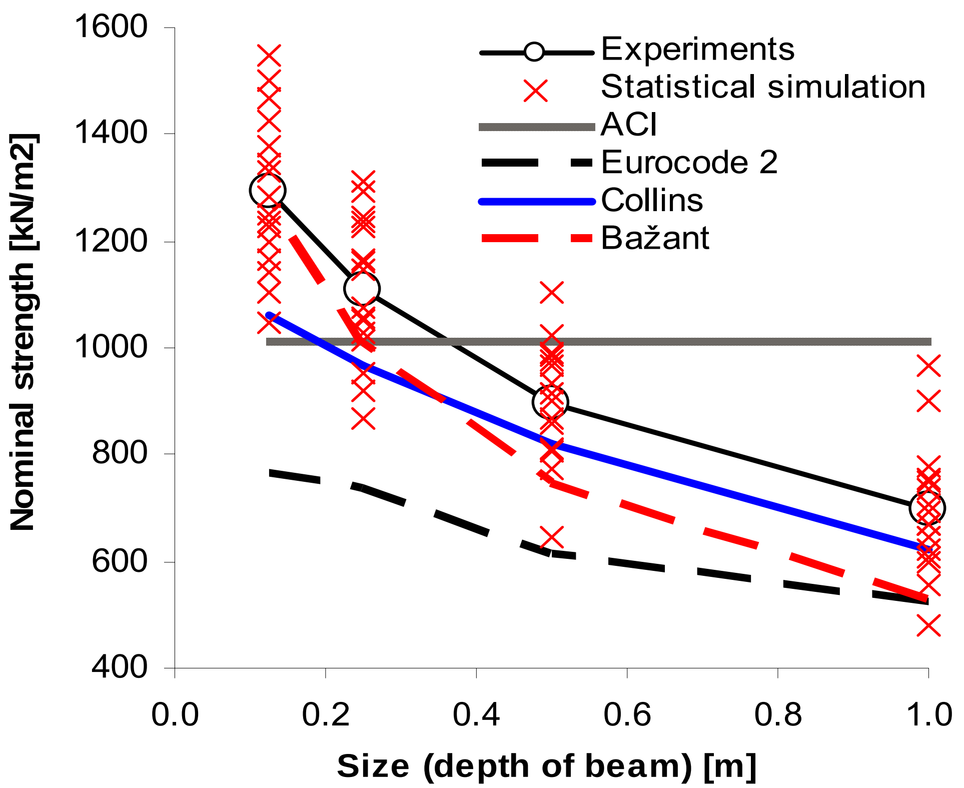
Size effect formulae and verification by statistical simulation



Verification of (code) design formulas: Shear failure of reinforced concrete beams





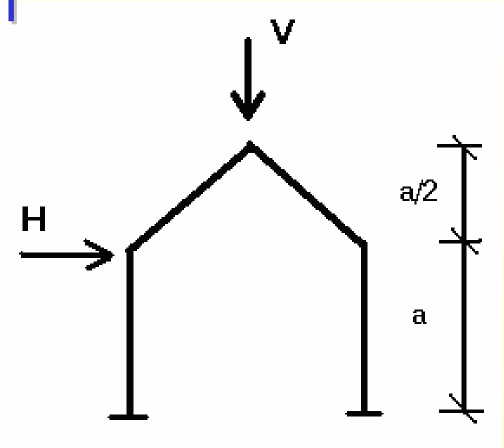


Nominal strength vs. size for different design alternatives: formulas, experiment and simulation

Inverse analysis

Training, stochastic preparation of training sets: classical Monte Carlo vs. Latin Hypercube Sampling methds

Failure surface approximmation



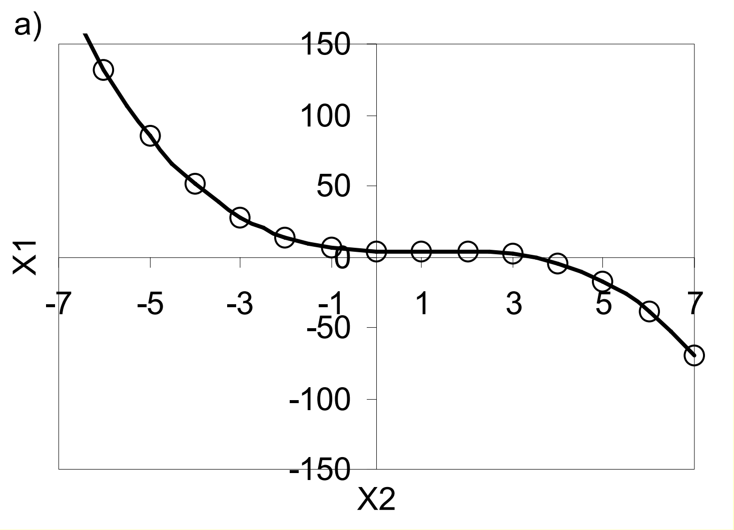
$$\boxed{g(\mathbf{X}) = aX_2^3 + bX_2^2 + cX_2 - X_1 + d}$$

$$a = -0{,}36355,\quad b = 1{,}18046,\quad c = -1.0892988,\quad d = 4.2042064$$

$$X_1 = Ha/m_p,\ X_2 = Va/m_p$$

Inverse analysis

Shape of failure surface:

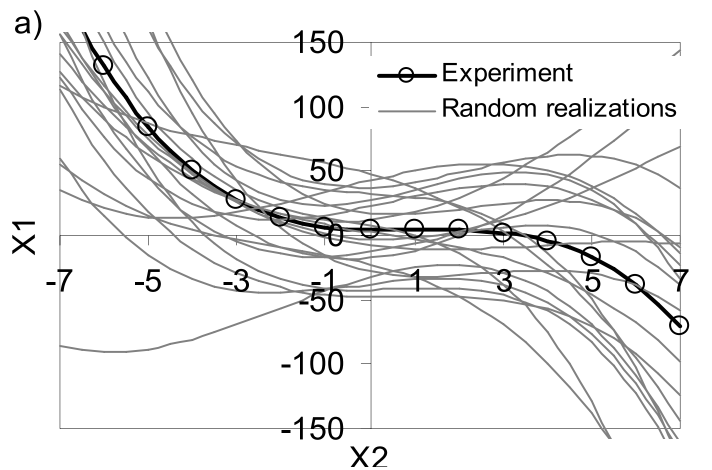


Aim: identification of parameters *a,b,c,d*

Parametric study for small numbers of simulations – 20,30,40 and 50

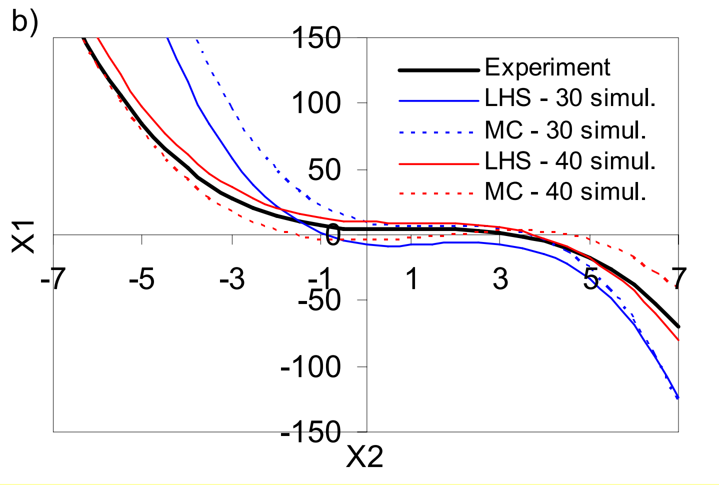
Same initial conditions (scattter of parameters, neural network type, same initiation of synaptic weights and biases to start training of network)

Inverse analysis

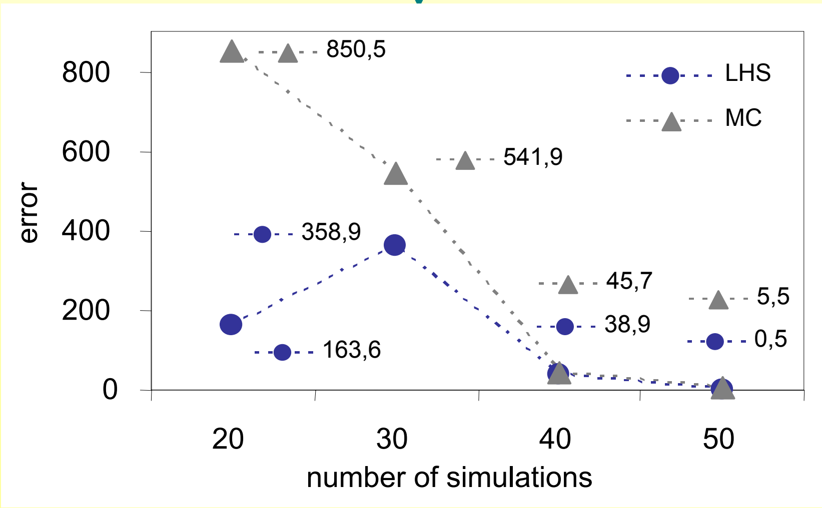


Training set (20 random realizations)

resulting failure surfaces



errors of identifications

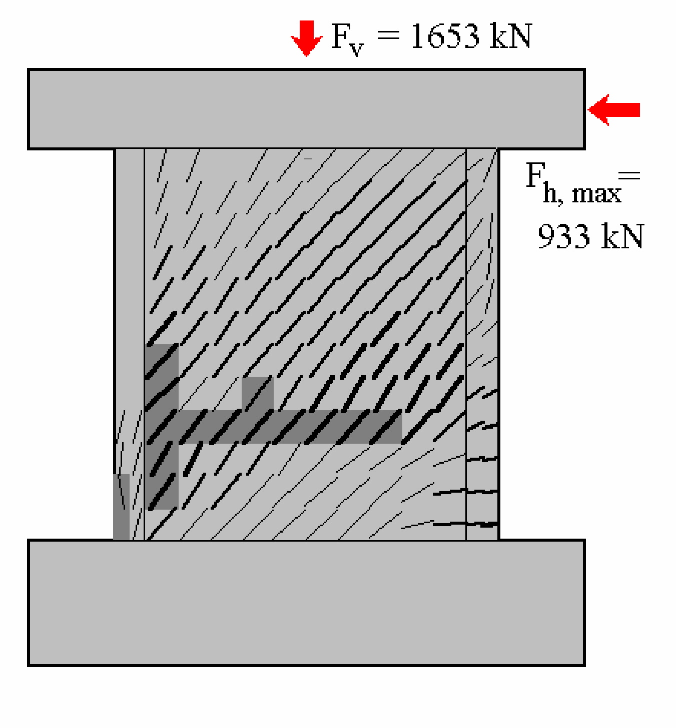


Identification of material parameters:
Shear wall test

Reality

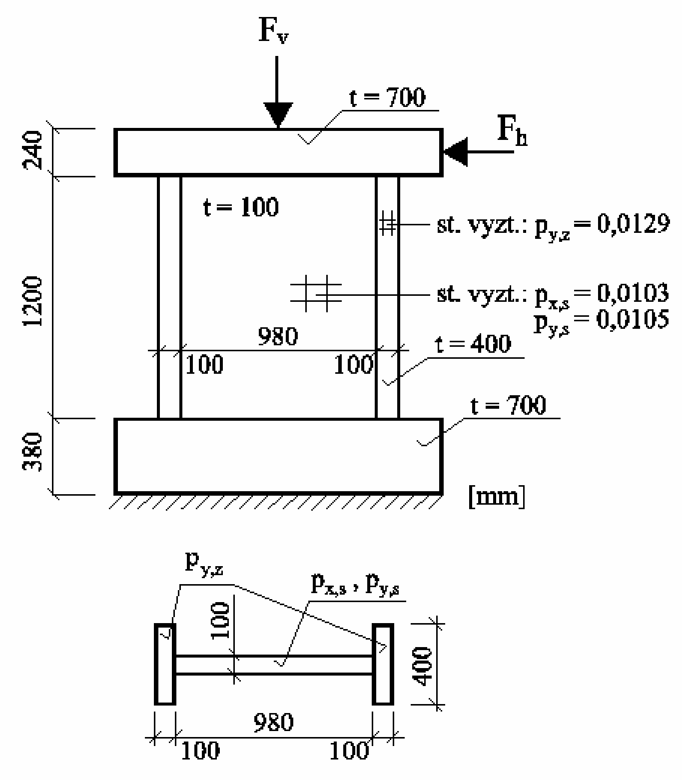
Experiments by
Maier and
Thürliman,
1985

Simulation



Identification of material parameters:

Shear wall test



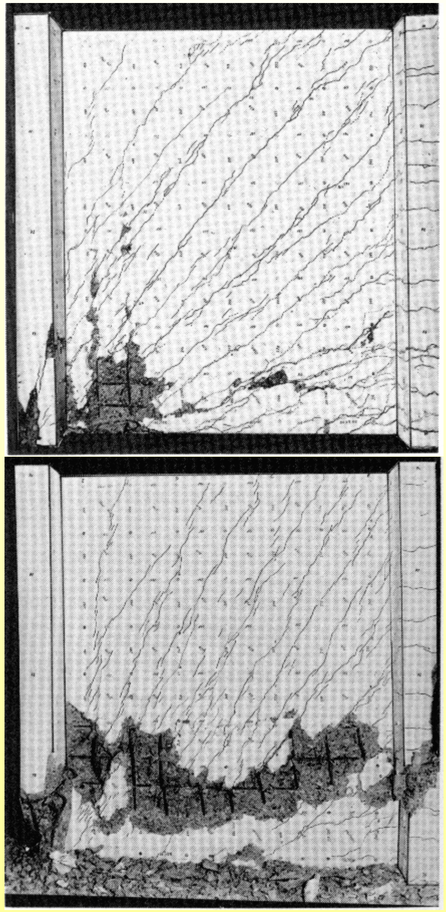
Variable	Symbol	Unit	Mean value	COV
Modulus of elasticity	E	GPa	30	0.10
Tensile strength	f_t	MPa	2.5	0.10
Compressive strength	f_c	MPa	30	0.10
Fracture energy	G_F	N/m	75	0.20
Compressive strain	ε_c	-	0.0025	0.20
Max. comp. displacement	w_d	m	0.003	0.30
Bilinear diagram of steel for smeared reinforcement	x_1	m	0.0027	0.10
	f_{x1}	kN	574	0.10
	x_2	m	0.015	0.10
	f_{x2}	kN	764	0.10

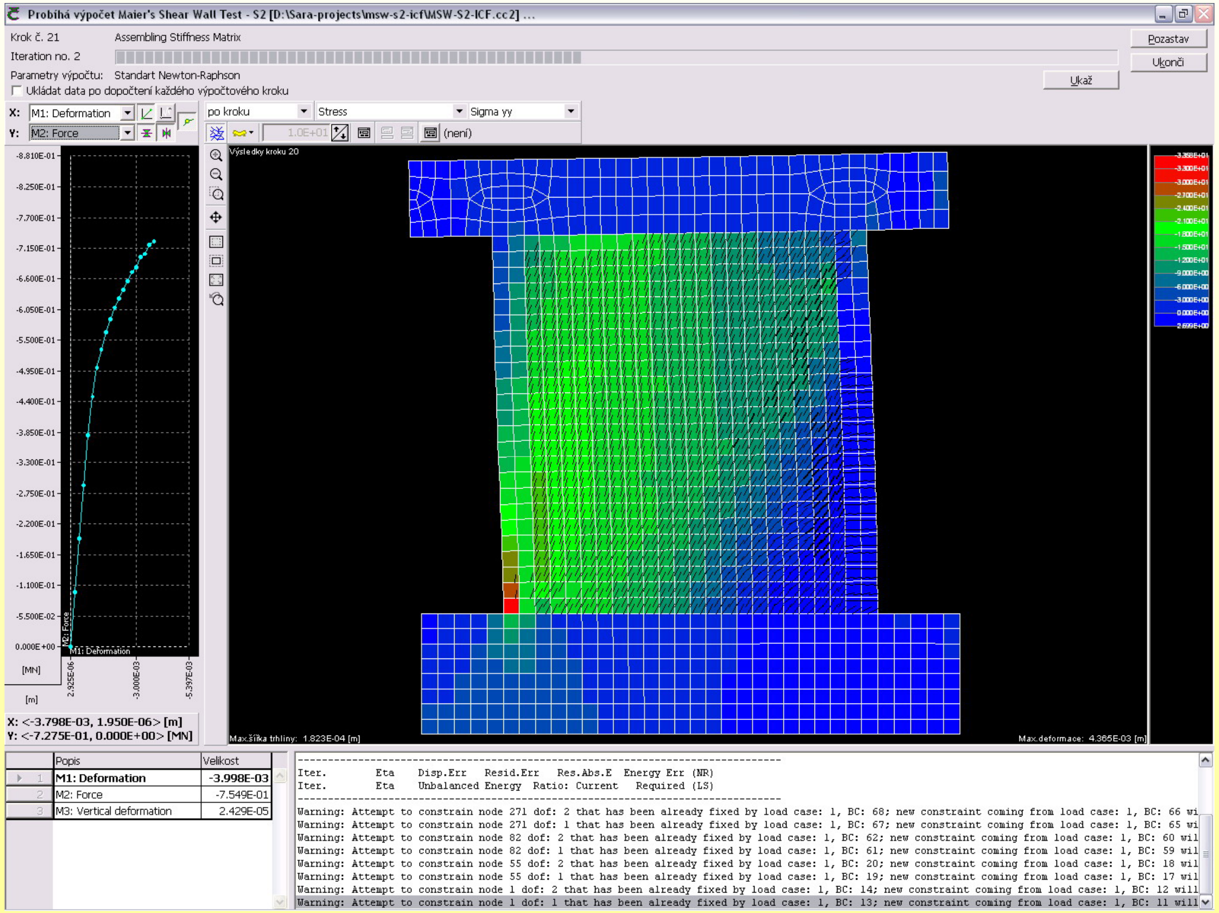
Randomization of material parameters – preparation of training set

Identification of material parameters:

Shear wall test

FEM model in software ATENA





Probíhá výpočet Maier's Shear Wall Test - S2 [D:\Sara-projects\msw-s2-icf\MSW-S2-ICF.cc2] ...

Krok č. 21 Assembling Stiffness Matrix

Iteration no. 2

Parametry výpočtu: Standart Newton-Raphson

☐ Ukládat data po dopočtení každého výpočtového kroku

X: M1: Deformation

Y: M2: Force

po kroku | Stress | Sigma yy

1.0E+01 (není)

Výsledky kroku 20

X: <-3.798E-03, 1.950E-06> [m]
Y: <-7.275E-01, 0.000E+00> [MN]

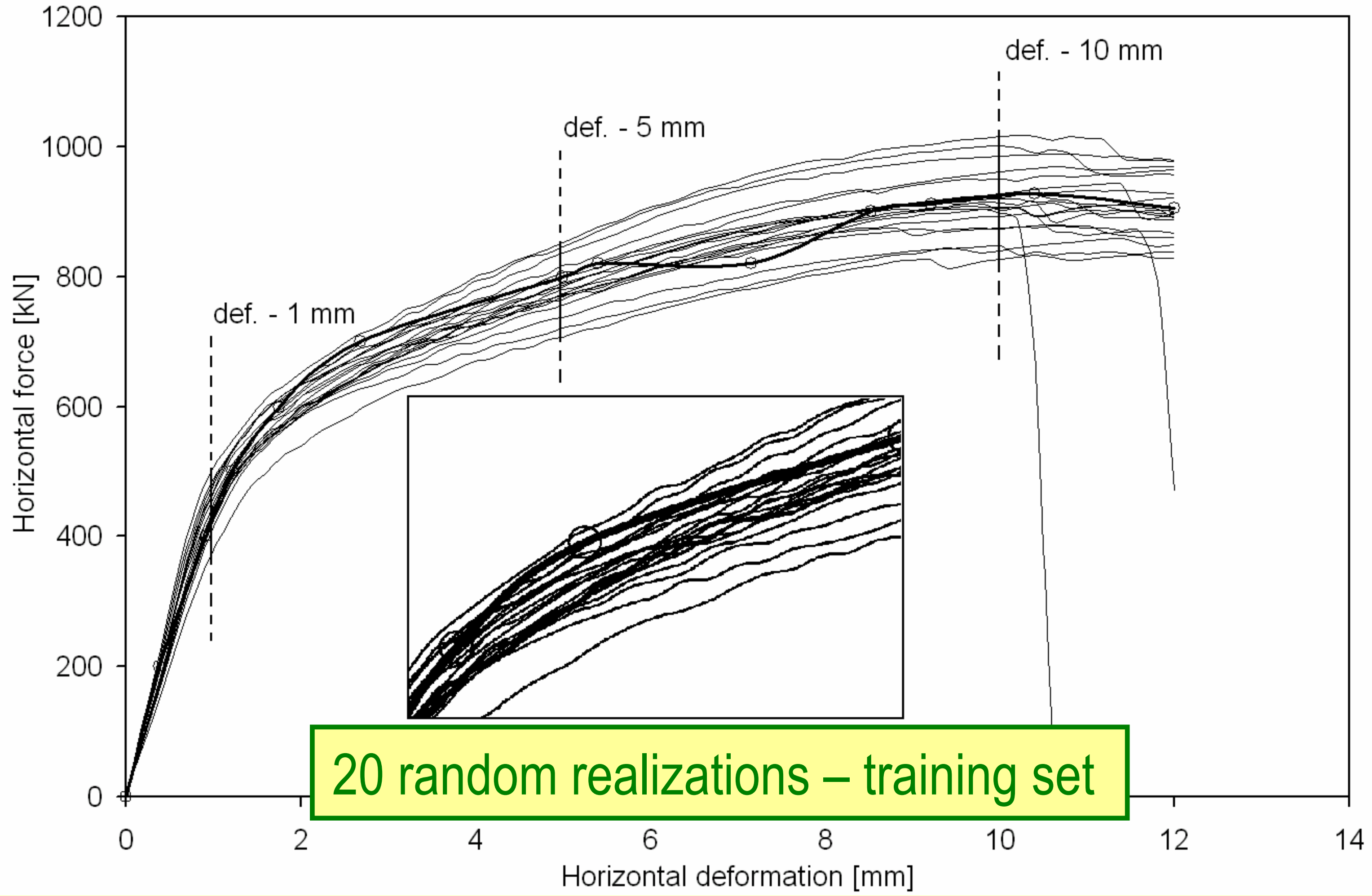
Max.šířka trhliny: 1.823E-04 [m]

Max.deformace: 4.365E-03 [m]

	Popis	Velikost
1	**M1: Deformation**	**-3.998E-03**
2	M2: Force	-7.549E-01
3	M3: Vertical deformation	2.429E-05

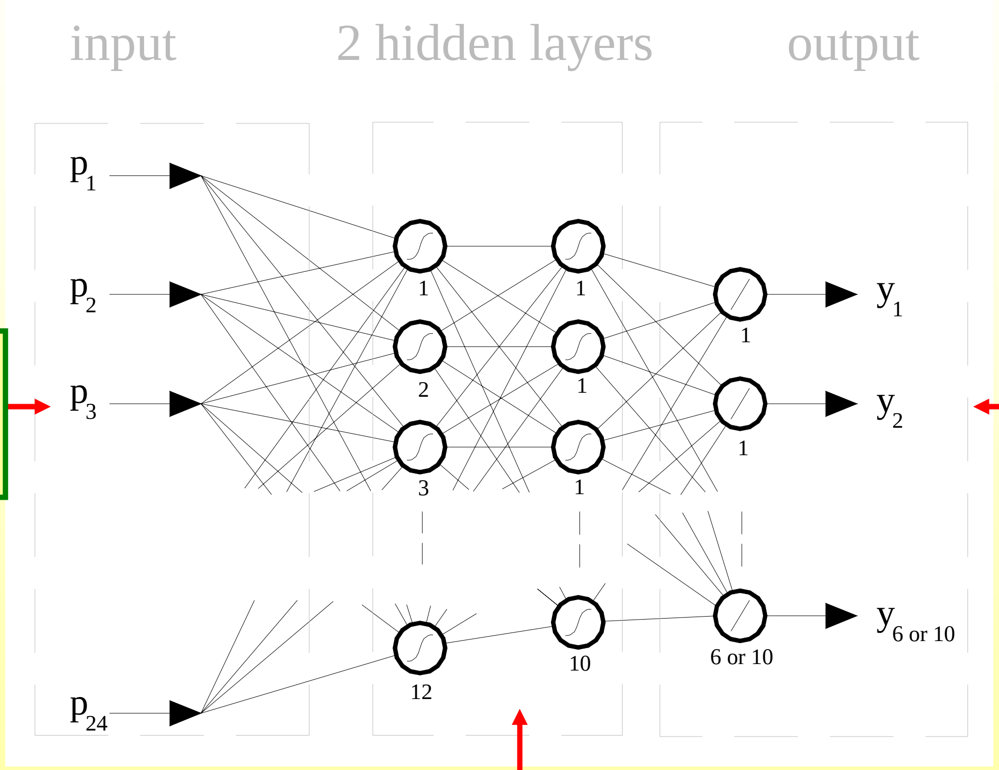
```
--------------------------------------------------------------------------------
Iter.        Eta    Disp.Err   Resid.Err   Res.Abs.E  Energy Err (NR)
Iter.        Eta    Unbalanced Energy  Ratio: Current   Required (LS)
--------------------------------------------------------------------------------
Warning: Attempt to constrain node 271 dof: 2 that has been already fixed by load case: 1, BC: 68; new constraint coming from load case: 1, BC: 66 wi
Warning: Attempt to constrain node 271 dof: 1 that has been already fixed by load case: 1, BC: 67; new constraint coming from load case: 1, BC: 65 wi
Warning: Attempt to constrain node 82 dof: 2 that has been already fixed by load case: 1, BC: 62; new constraint coming from load case: 1, BC: 60 wil
Warning: Attempt to constrain node 82 dof: 1 that has been already fixed by load case: 1, BC: 61; new constraint coming from load case: 1, BC: 59 wil
Warning: Attempt to constrain node 55 dof: 2 that has been already fixed by load case: 1, BC: 20; new constraint coming from load case: 1, BC: 18 wil
Warning: Attempt to constrain node 55 dof: 1 that has been already fixed by load case: 1, BC: 19; new constraint coming from load case: 1, BC: 17 wil
Warning: Attempt to constrain node 1 dof: 2 that has been already fixed by load case: 1, BC: 14; new constraint coming from load case: 1, BC: 12 will
Warning: Attempt to constrain node 1 dof: 1 that has been already fixed by load case: 1, BC: 13; new constraint coming from load case: 1, BC: 11 will
```

Identification of material parameters



20 random realizations – training set

Identification of material parameters



24 points on
l-d diagram

6 or 10 neurons
in output layer
– linear transfer
function

12 and 10 neurons in 2 hidden layers
– nonlinear transfer function

Identification of material parameters

Spearman	E	f_t	f_c	G_f	ε_c	w_d	x_1	fx_1	x_2	fx_2
F_1	0,753	0,123	0,453	0,045	-0,335	-0,108	-0,167	0,015	-0,087	-0,107
F_5	0,262	0,513	0,460	0,014	-0,263	-0,081	-0,516	0,311	-0,051	0,045
F_{10}	0,158	0,382	0,608	0,081	-0,080	-0,027	-0,344	0,490	0,005	0,104
F_{max}	0,129	0,341	0,636	0,054	-0,042	-0,053	-0,307	0,537	-0,009	0,171

Sensitivity of material model parameters:

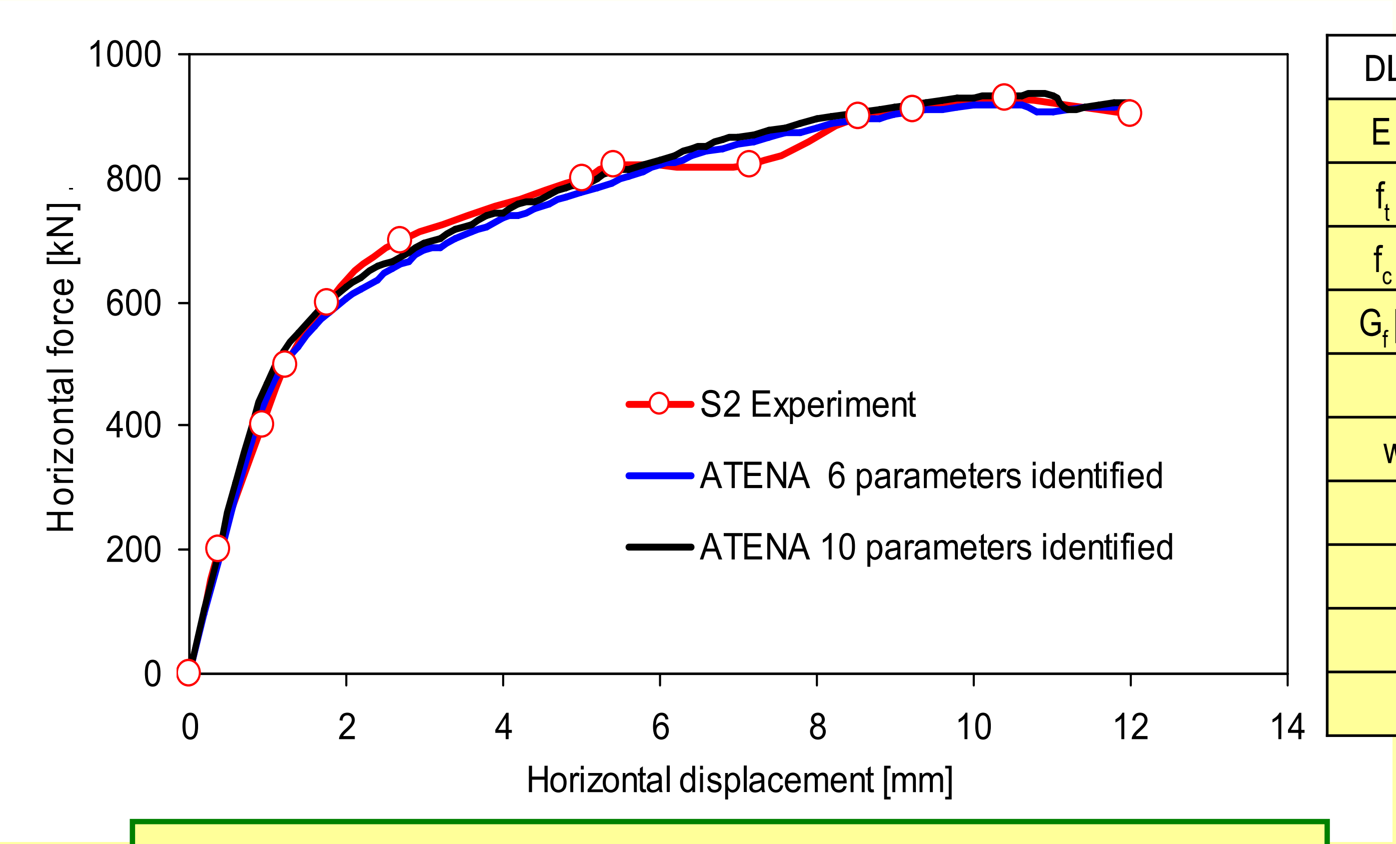
- 6 parameters identified
- 10 parameters identified

Parameters obtained from simulation of neural network:

DLNNET	*6 par.*	*10 par.*
E [MPa]	29,9	33,0
f_t [MPa]	2,47	2,47
f_c [MPa]	34,51	35,3
G_f [MN/m]	*75,0*	77,85
e_c [-]	2,51E-03	2,57E-03
w_d [m]	*3,00E-03*	3,10E-03
x_1	2,72E-03	2,74E-03
fx_1	566,9	570,7
x_2	*1,50E-02*	1,47E-02
fx_2	*764*	768,8

Identification of material parameters:

Shear wall test

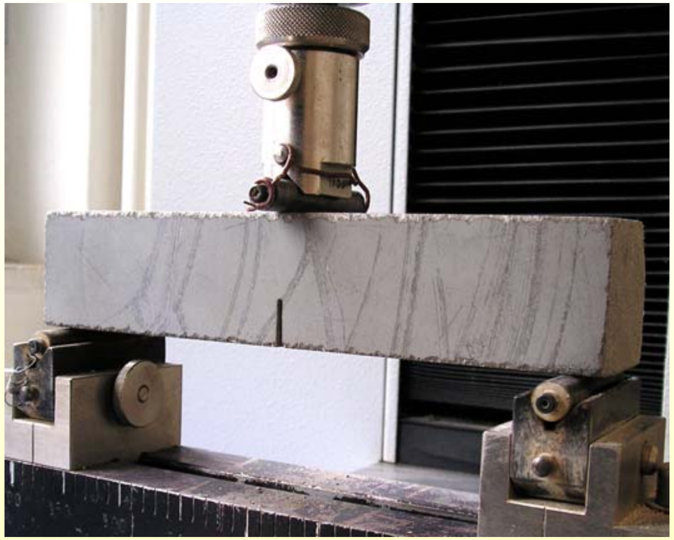


DLNNET	*6 par.*	*10 par.*
E [MPa]	29,9	33,0
f_t [MPa]	2,47	2,47
f_c [MPa]	34,51	35,3
G_f [MN/m]	*75,0*	77,85
e_c [-]	2,51E-03	2,57E-03
w_d [m]	*3,00E-03*	3,10E-03
x_1	2,72E-03	2,74E-03
fx_1	566,9	570,7
x_2	*1,50E-02*	1,47E-02
fx_2	*764*	768,8

L-d diagrams obtained with identified parameters

Experimental works (VUSTAH)

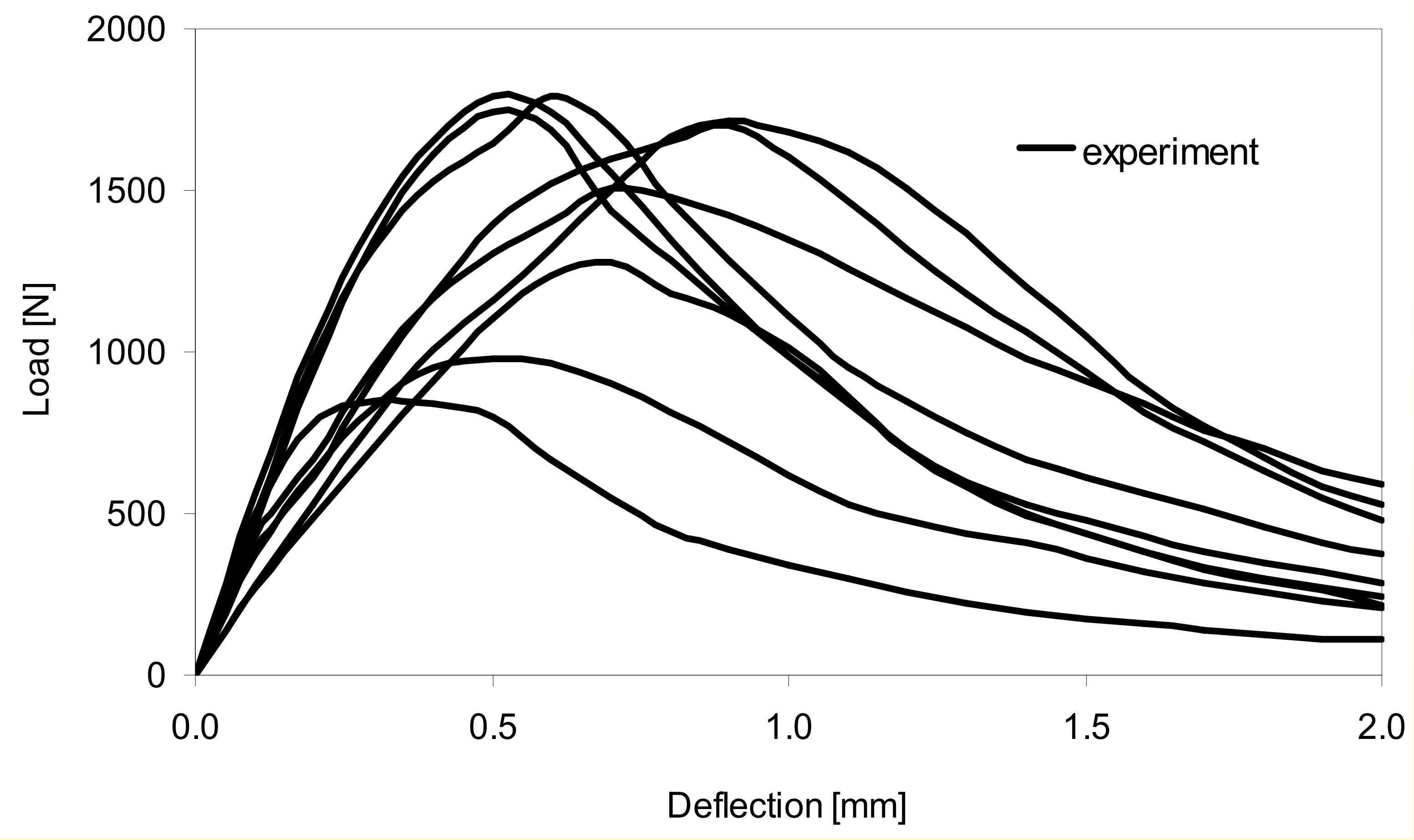
3-point bending experiment of fibre-reinforced concrete notched beams



Specimen parameter	Units	Value
Length of specimen	mm	200
Width of specimen	mm	40
Depth of specimen	mm	40
Depth of notch	mm	15
Weight	kg	0,67
Span	mm	180

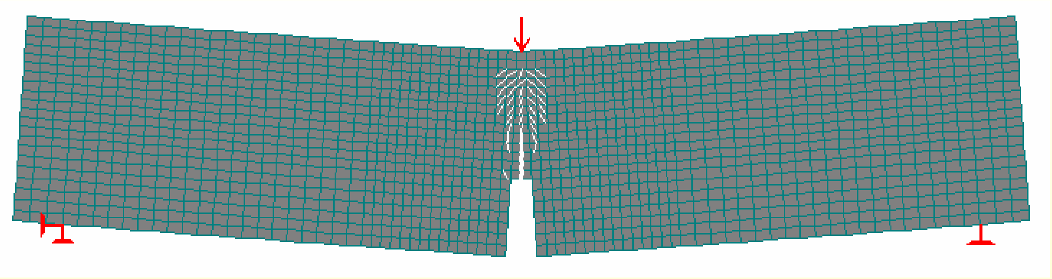
Experimental works (VUSTAH)

9 experimental load-deflection curves



Virtual numerical simulation of experiment

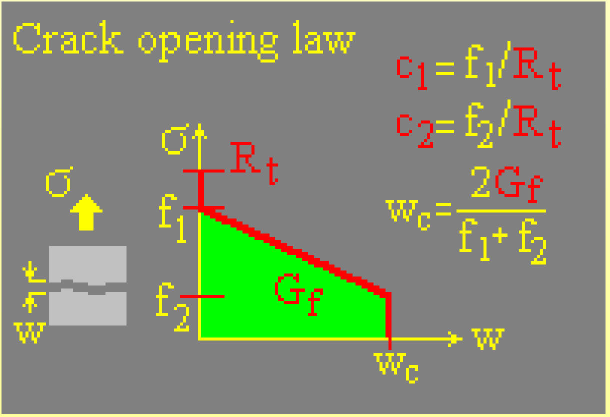
Nonlinear analysis – software ATENA (Červenka Consulting



Material model SBETA – nonlinear fracture mechanics:

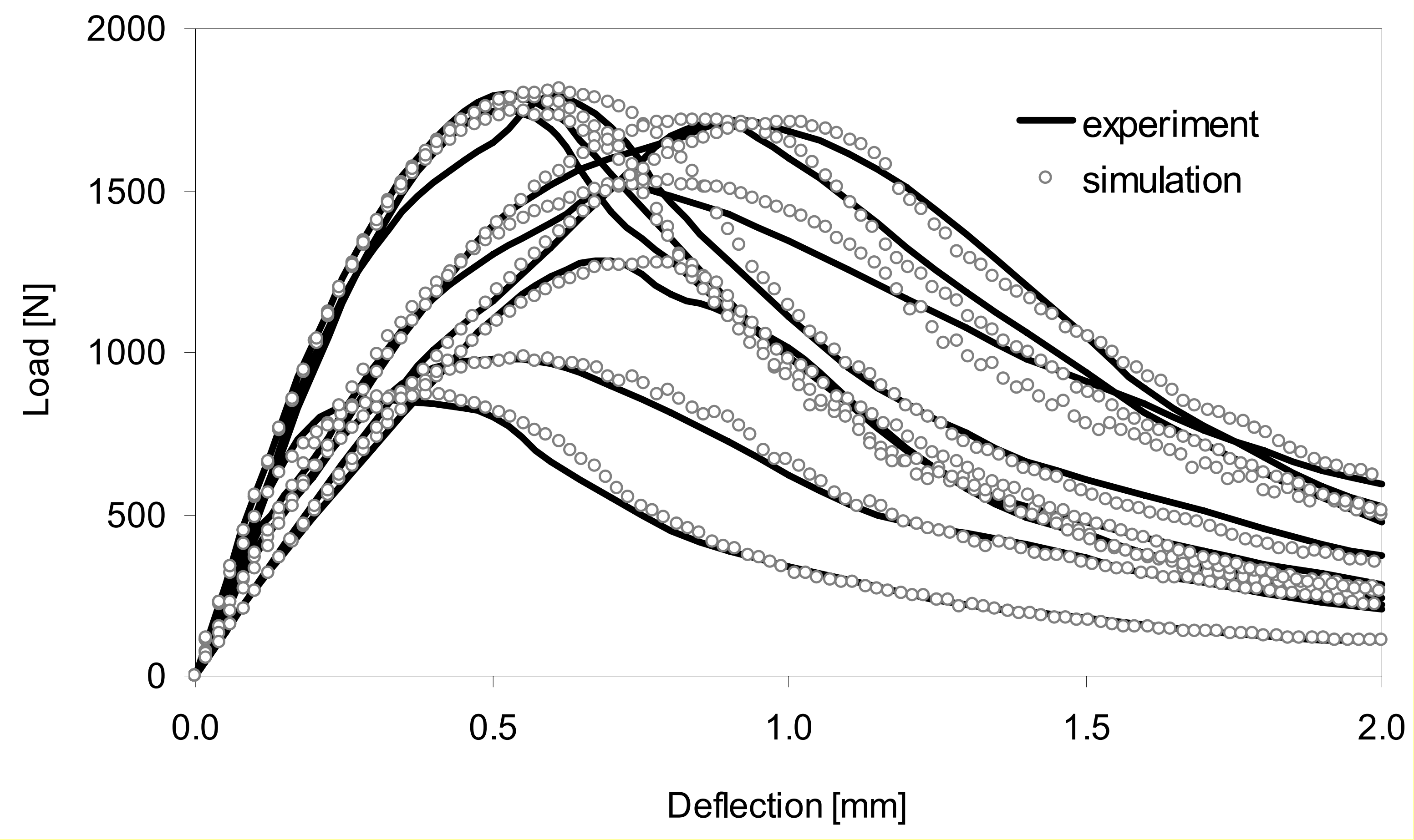
- smeared cracks model (fixed or rotated cracks)
- crack band method (localization limiter)
- crack opening ⇔ fracture energy
- softening model for fibre-reinforced concrete

(parameters c_1 and c_2)



Inverse analysis

Experimental + virtual (numerical) load-deflection curves



Statistics of fracture-mechanical parameters

Inverse analysis based on neural networks

Parameter	Unit	Mean value	Standard deviation	Coefficient of variation in %
Maximum failureload	kN	1,49	0,36	24,1
Deflection a maximum load	mm	0,67	0,20	29,3
Moduus of elasticity	GPa	5,4	1,68	30,9
Tensile strength	MPa	11,3	3,39	29,9
Fracture energy	J/m^2	2134	673	31,5
Softening parametr c_1	-	0,9	0,02	2,5
Softening parametr c_2	-	0,1	-	-

Conclusions

- Methods for statistical, sensitivity and reliability analyses, suitable for analysis of computationally intensive problems (eg. continuum mechanics, FEM)
- Software tools FREET and SARA - for the assessment of real behavior of concrete structures, can be applied for any problem of quasibrittle modeling of concrete structures
- A wide range of applicability both practical and theoretical - gives an opportunity for further intensive development of both methods and software