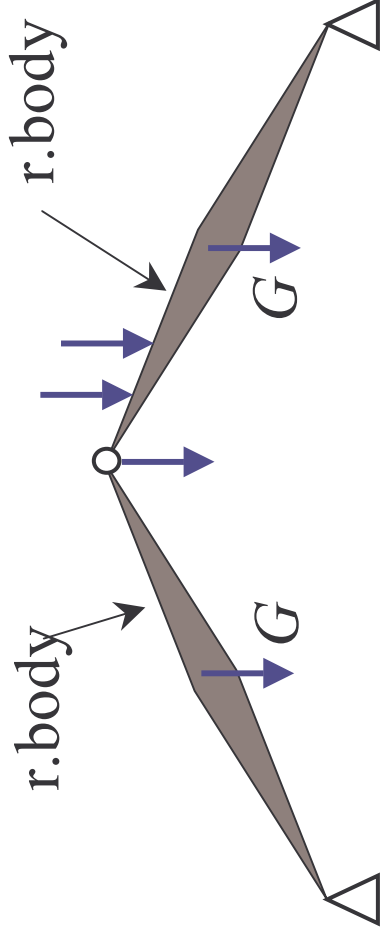


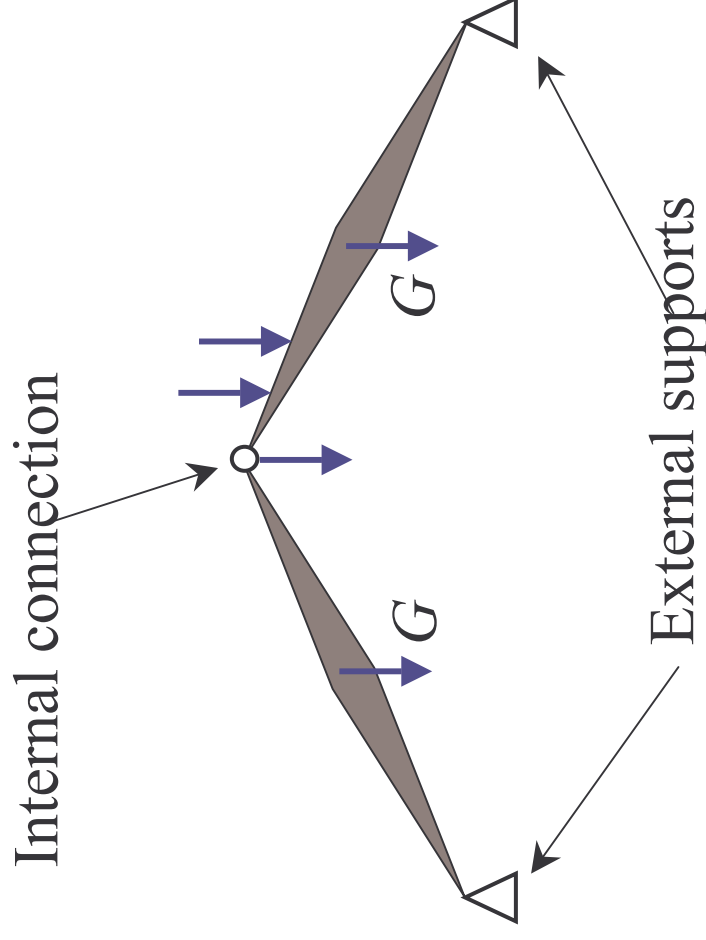
## 4.5 Composed structures

- Are created by combining several structural members (rigid particles, rigid bodies)



- supports, connections:

- internal – connect individual members
- external – connect the member to the ground



- degrees of freedom

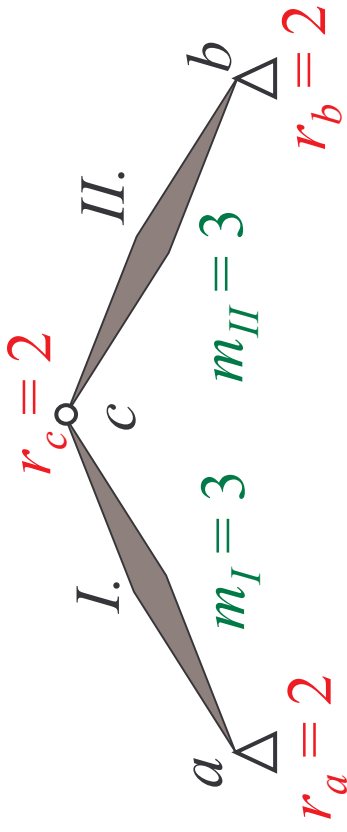
- $m$  = sum of degrees of freedom of individual members
- $r$  = sum of constrained degrees of freedom
- $s = m - r$  number of degrees of freedom of the structural system

- Statically determinate/indeterminate structures

- overall – check all members and all supports (external and internal)
- external – check only the whole structure (2D:  $m_{external} = 3$ , 3D:  $m_{external} = 6$ ) and external supports

Ex.:

➤ overall check



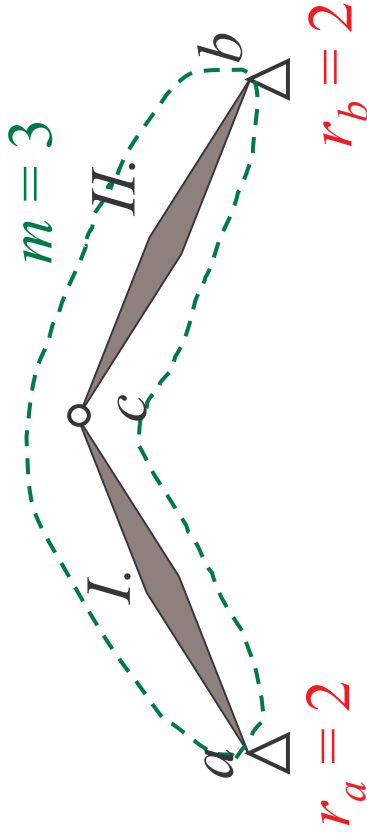
$$m = 2 \times 3 = 6$$

$$r = 3 \times 2 = 6$$

$$m = r \dots s = 0$$

statically determinate from the overall point of view

➤ external check



$$m_{\text{external}} = 3$$

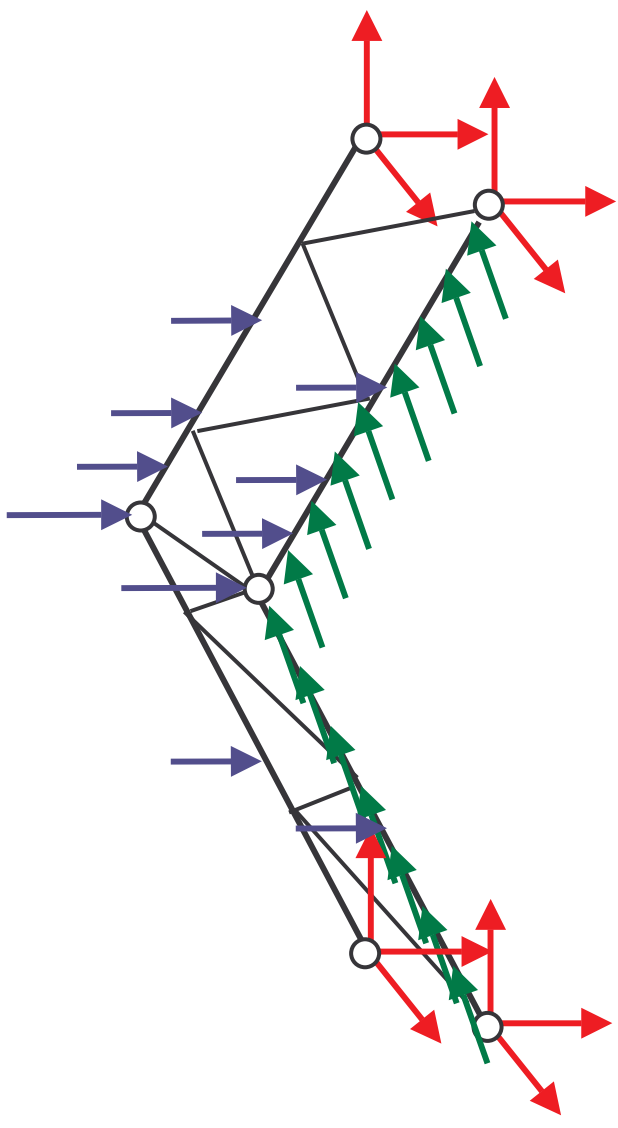
$$r_{\text{external}} = 2 + 2 = 4$$

$$m_{\text{external}} < r_{\text{external}} \dots s_{\text{external}} = -1$$

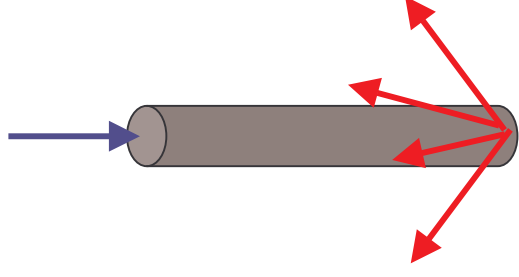
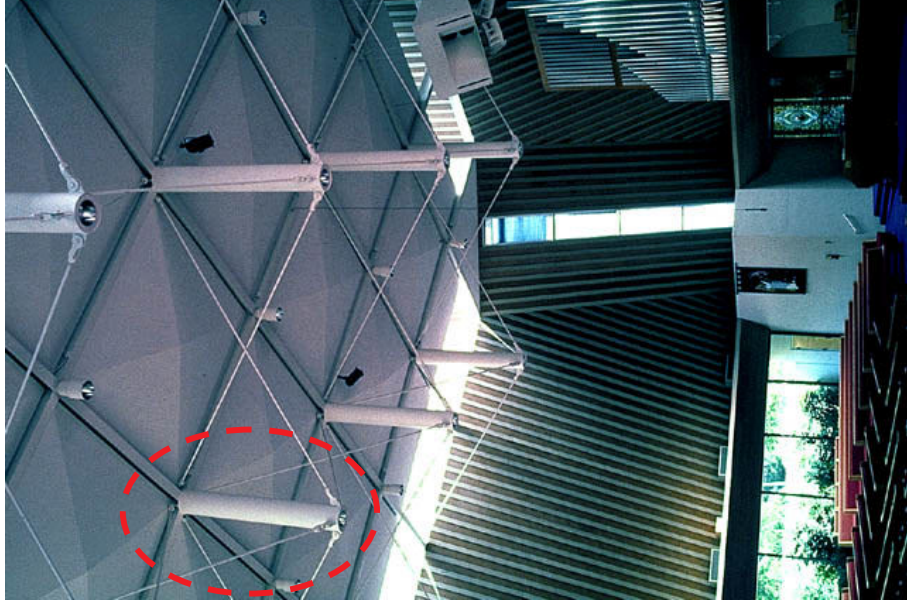
statically determinate from the external point of view

- space structure

- loading: general space system of forces **or**
- 3D arrangement of supports



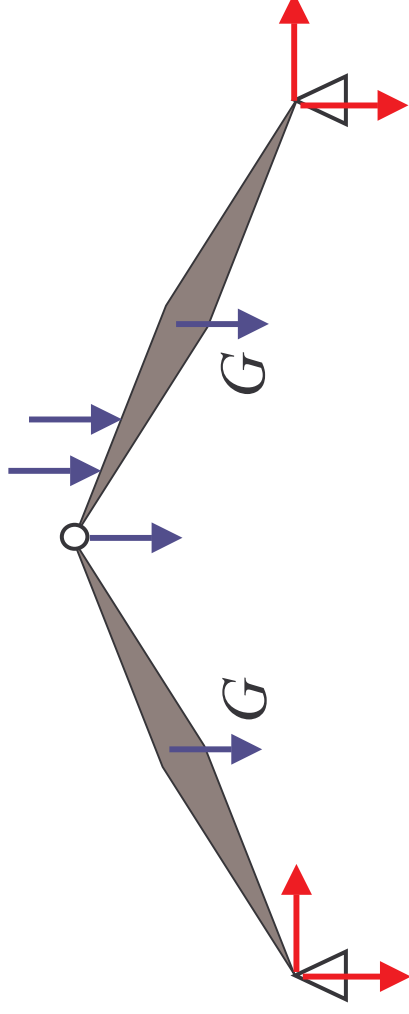
- space structure



*foto: Godden Structural Engineering Slide Library  
Courtesy National Information Service for Earthquake Engineering,  
University of California, Berkeley*

- plane structure

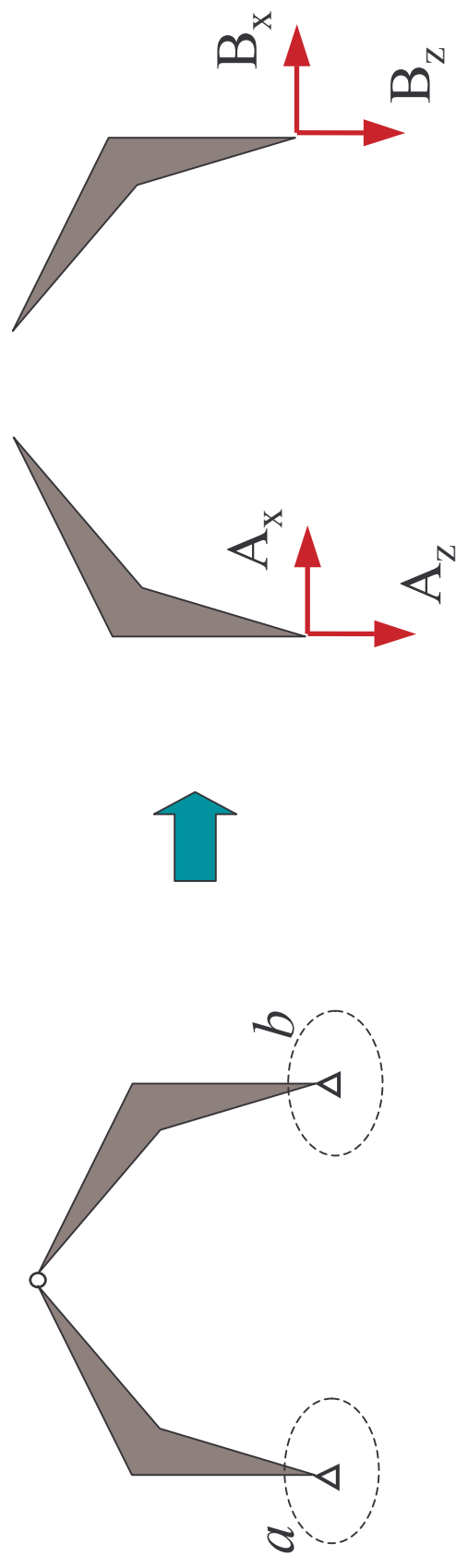
- loading: 2D system of forces **and**
- 2D arrangement of supports



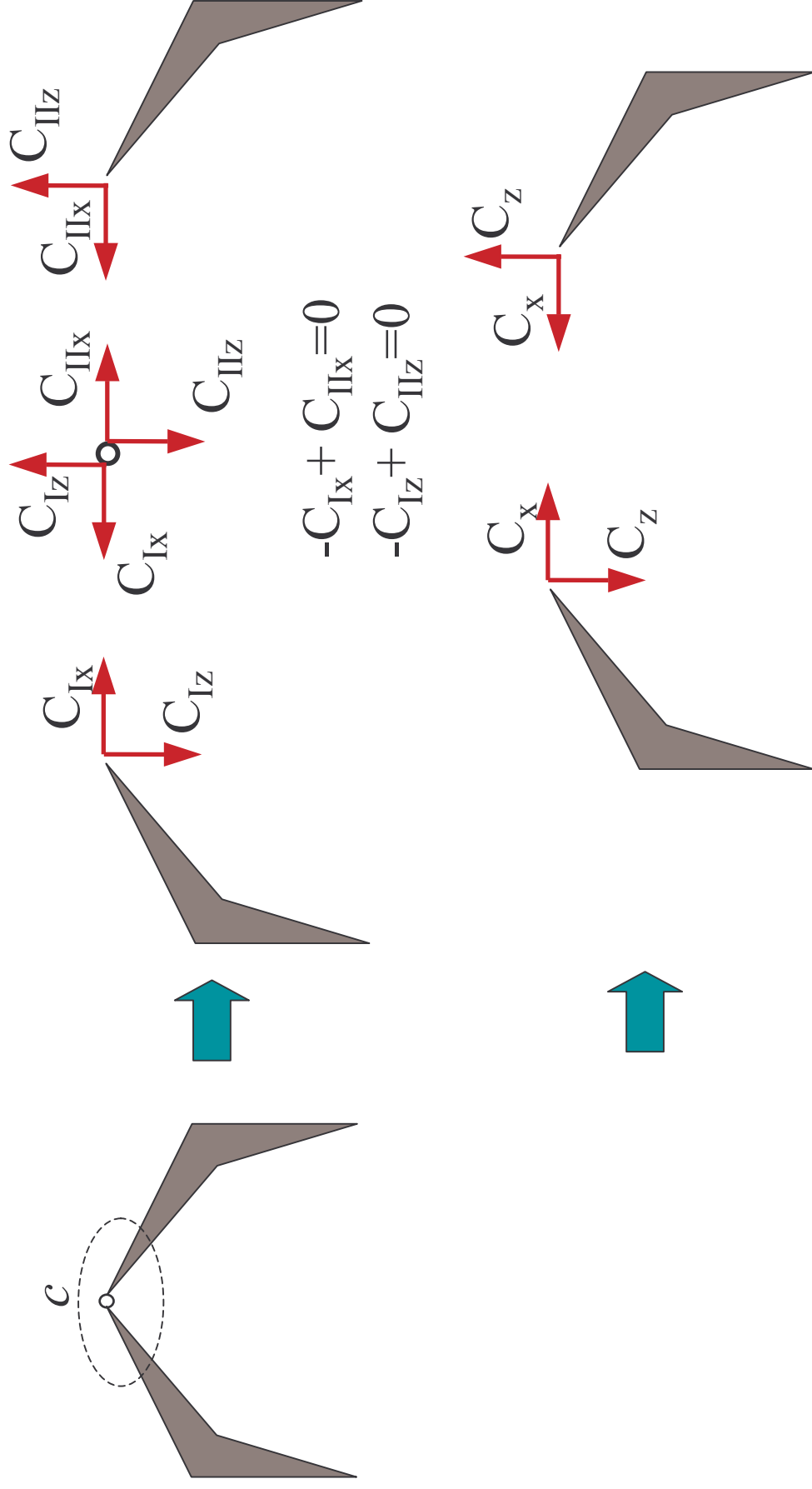
- method of computing reaction forces:

- 1) Separate individual members of a structure
- 2) Replace supports by reaction forces

- external supports



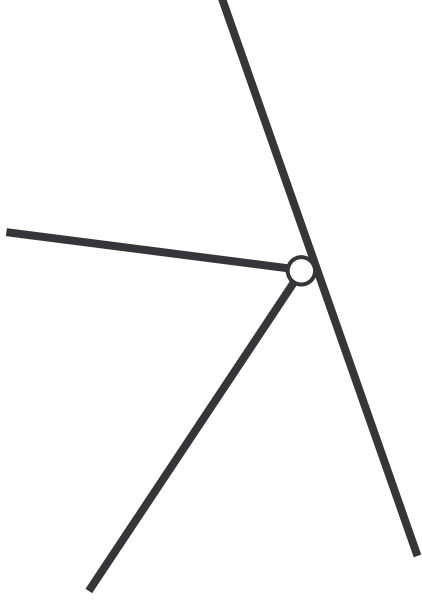
- internal supports: each internal support must be in equilibrium



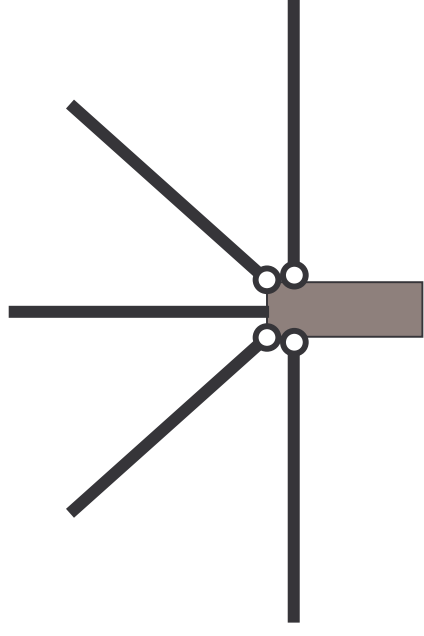
- 3) The reactions are found such that each part of the structure is in the state of equilibrium  $\Leftrightarrow$  the whole structure is in the state of equilibrium
- Equations of equilibrium for each structural member
    - include only reactions and loading pertinent to a given member (free body diagram)
  - Global (external) equations of equilibrium for the whole structure
    - include all external (applied) loading and external reactions
    - redundant equations of equilibrium (can be used to check the previous computation)

## 4.5.1 2D composed structures

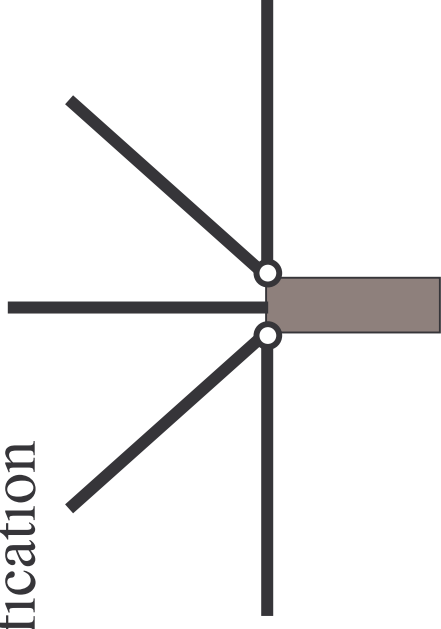
- internal hinge connecting more than two structural members



- internal hinge connecting n members



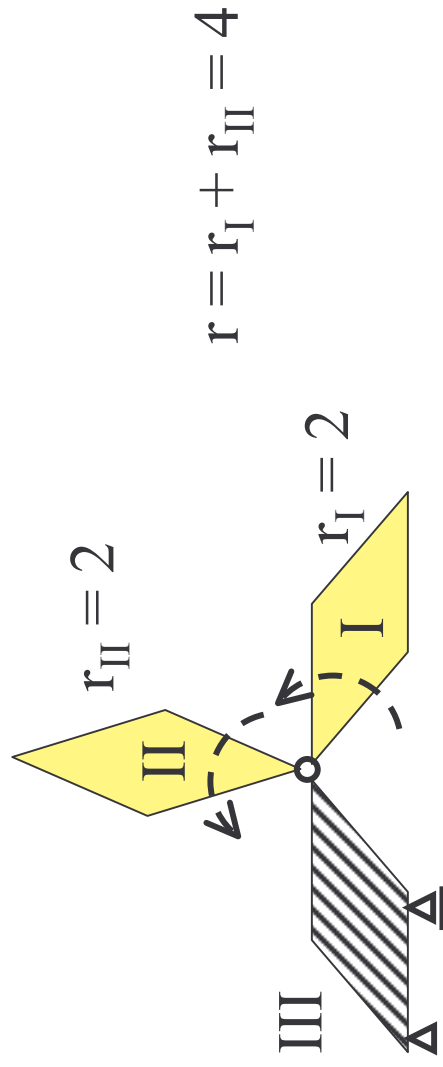
For computation of reaction affecting Connected elements we accept sometimes simplification



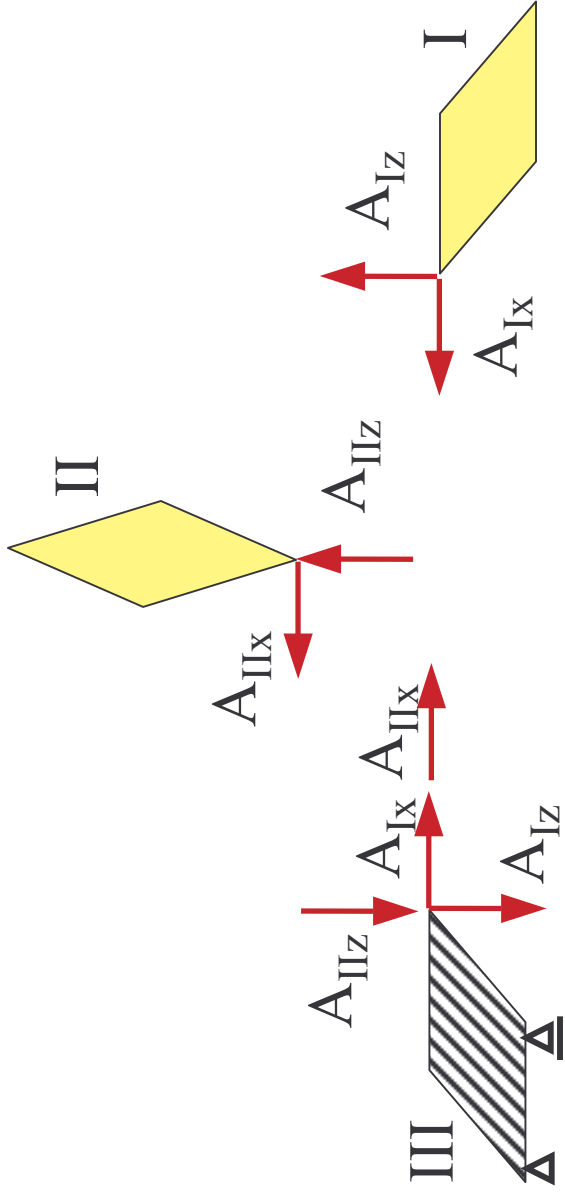
*warning: not applicable for detailed analysis of a support.*

\* Removed dofs. :  $r = 2 (n-1)$

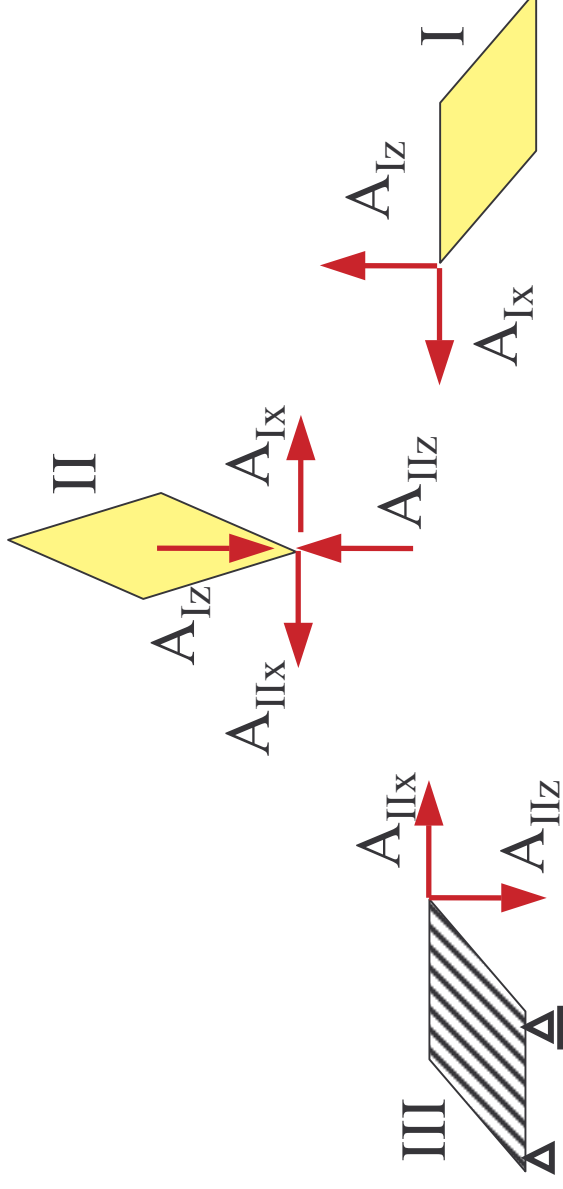
(3 rigid bodies connected by an internal hinge)



\* reactions



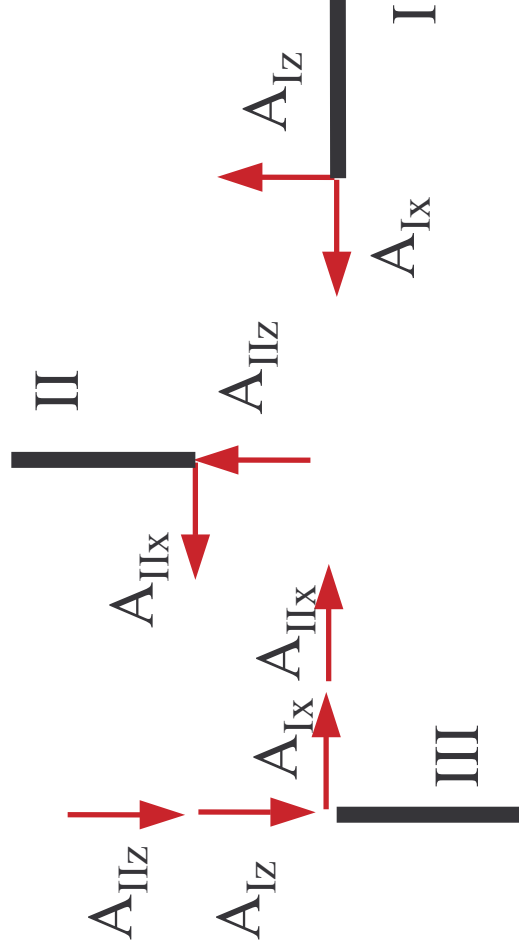
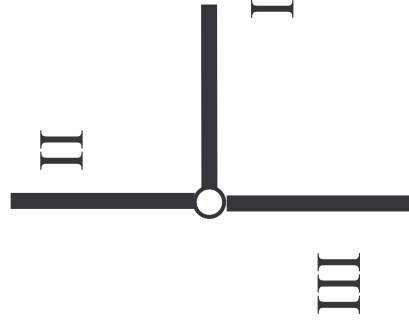
or



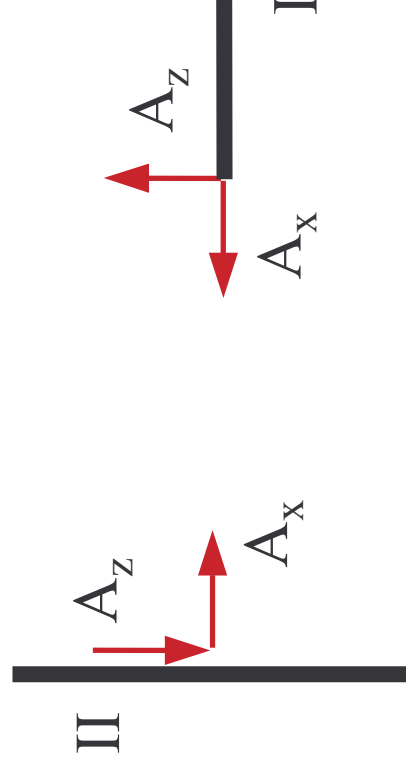
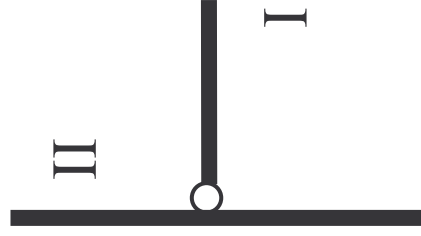
*warning: not applicable for detailed analysis of a support. In such a case all reactions have to be specified according to the detailed drawing of a given support!*

\* warning, difference

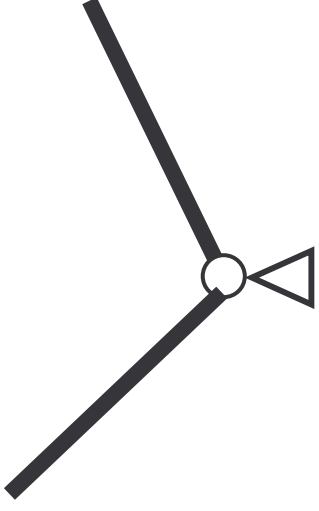
*3-member hinge*



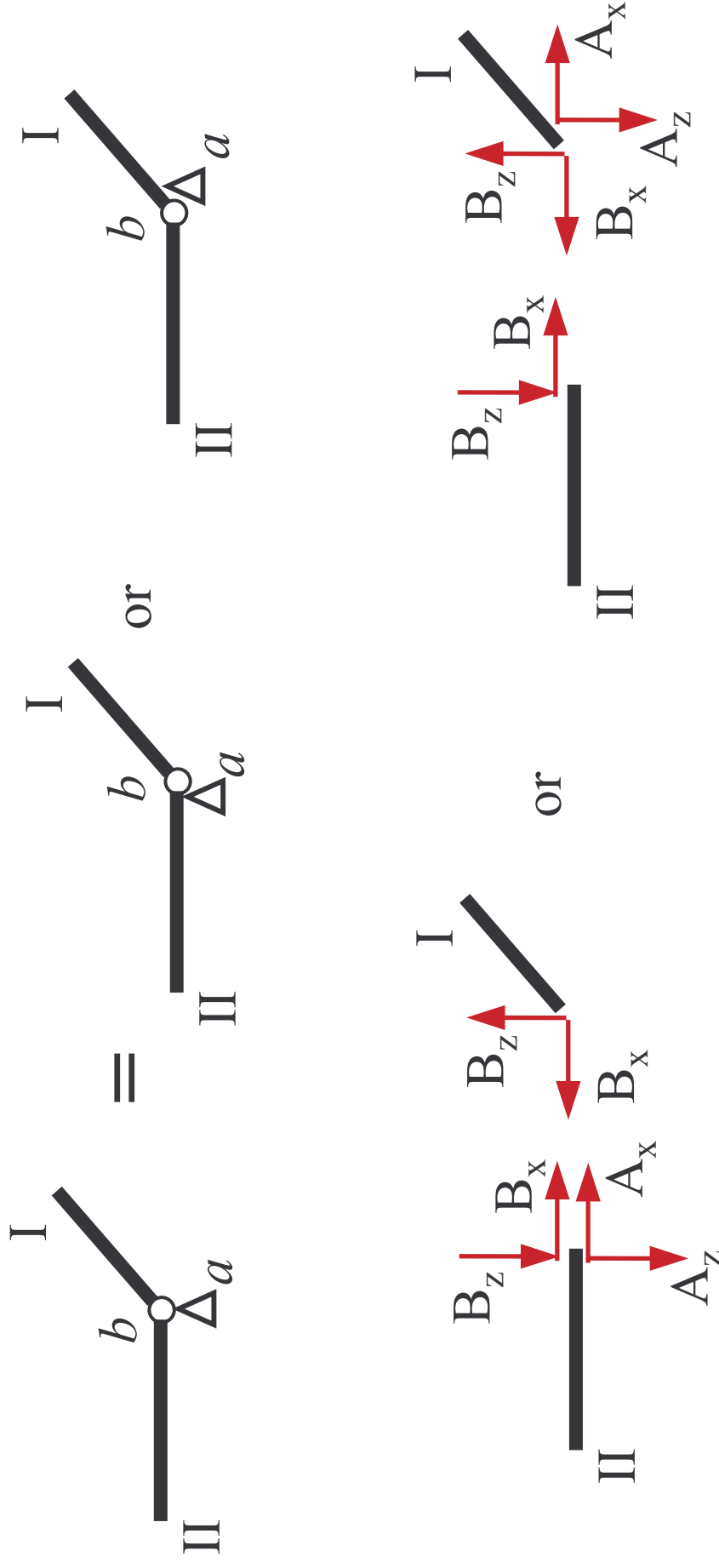
*simple hinge*



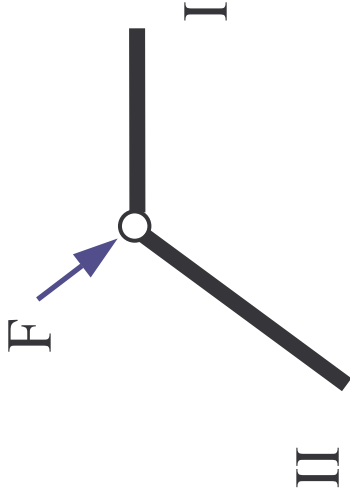
- two or more supports in one place, e.g.:



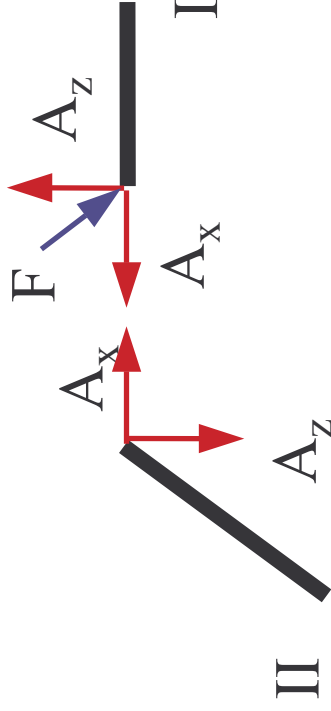
- two or more supports in one place, e.g.:



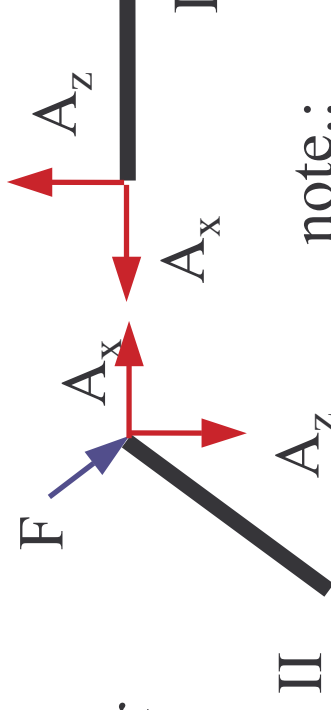
- loaded support, e.g.:



The external force can be assigned  
either to body #I or #II



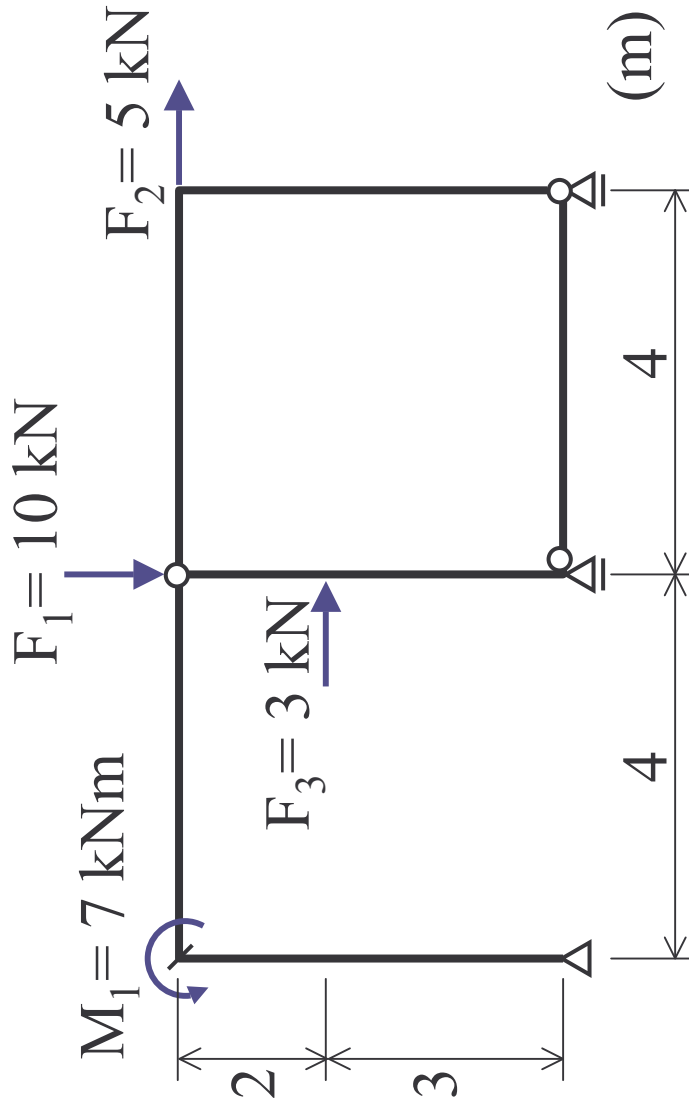
or



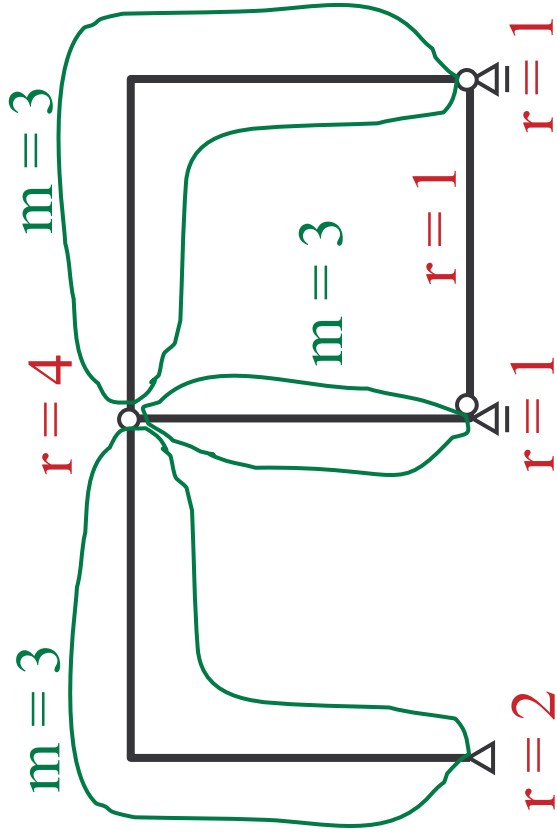
note.:

Similarly for moment

Ex.: Determine external and internal reactions



Determine s

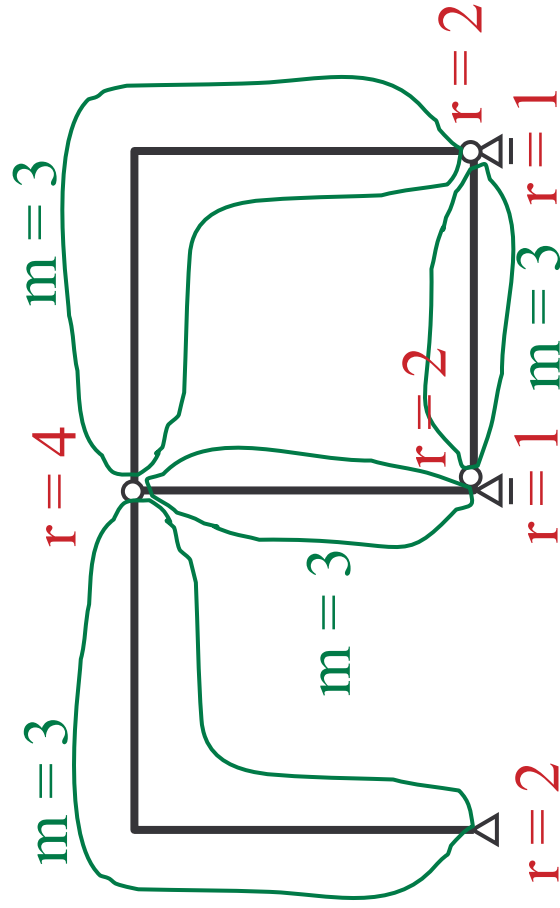


$$m = 3 \times 3 = 9$$

$$r = 3 \times 1 + 2 + 4 = 9$$

$$m = r$$

or

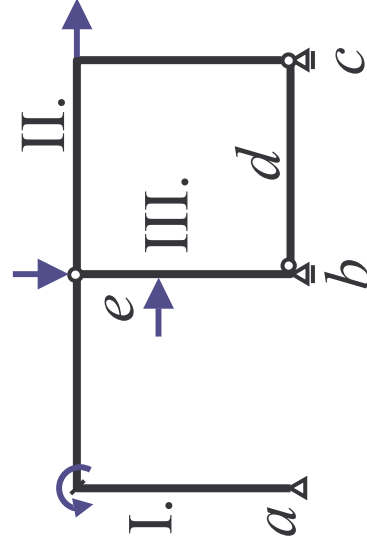
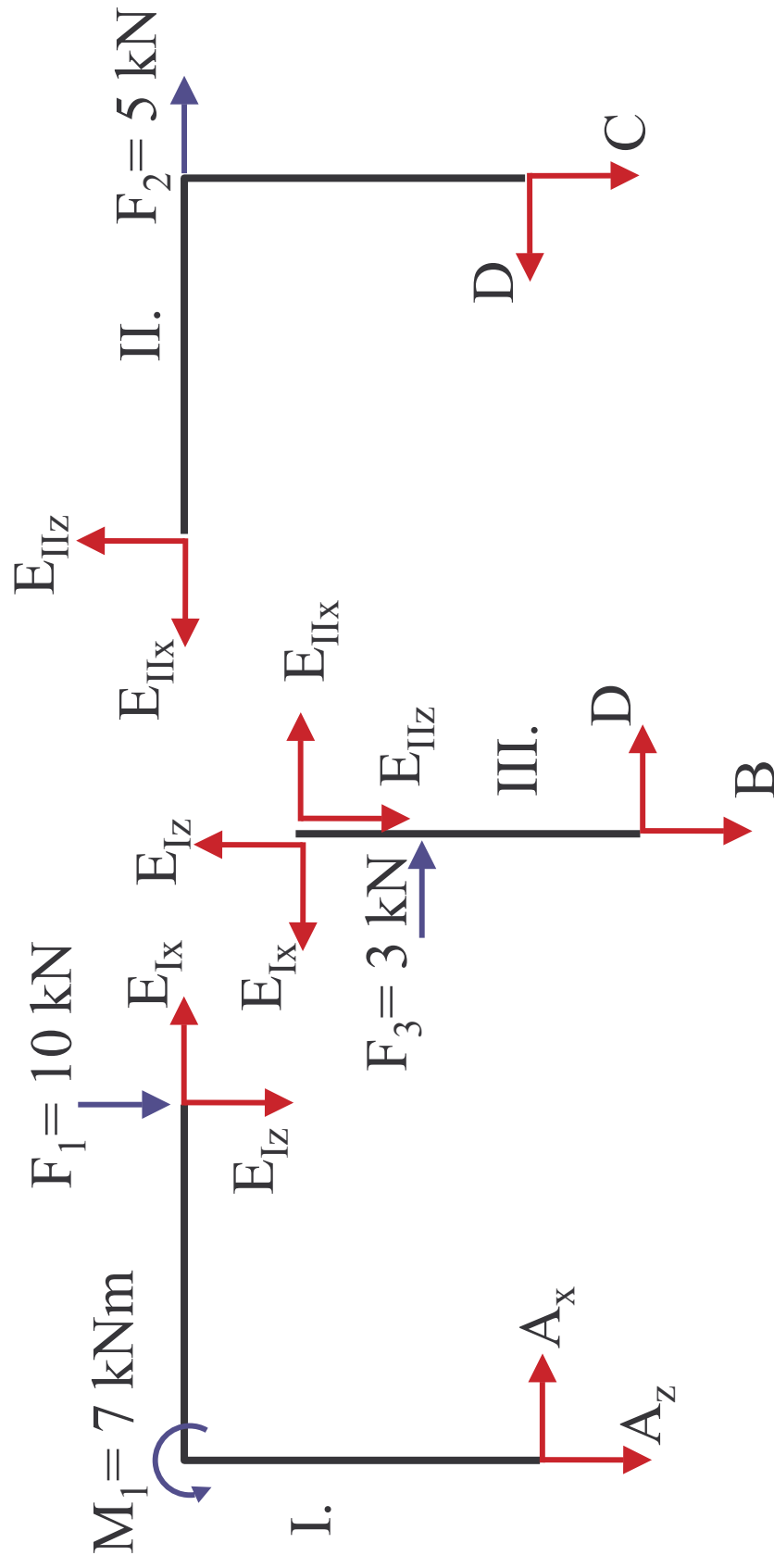


$$m = 4 \times 3 = 12$$

$$r = 2 \times 1 + 3 \times 2 + 4 = 12$$

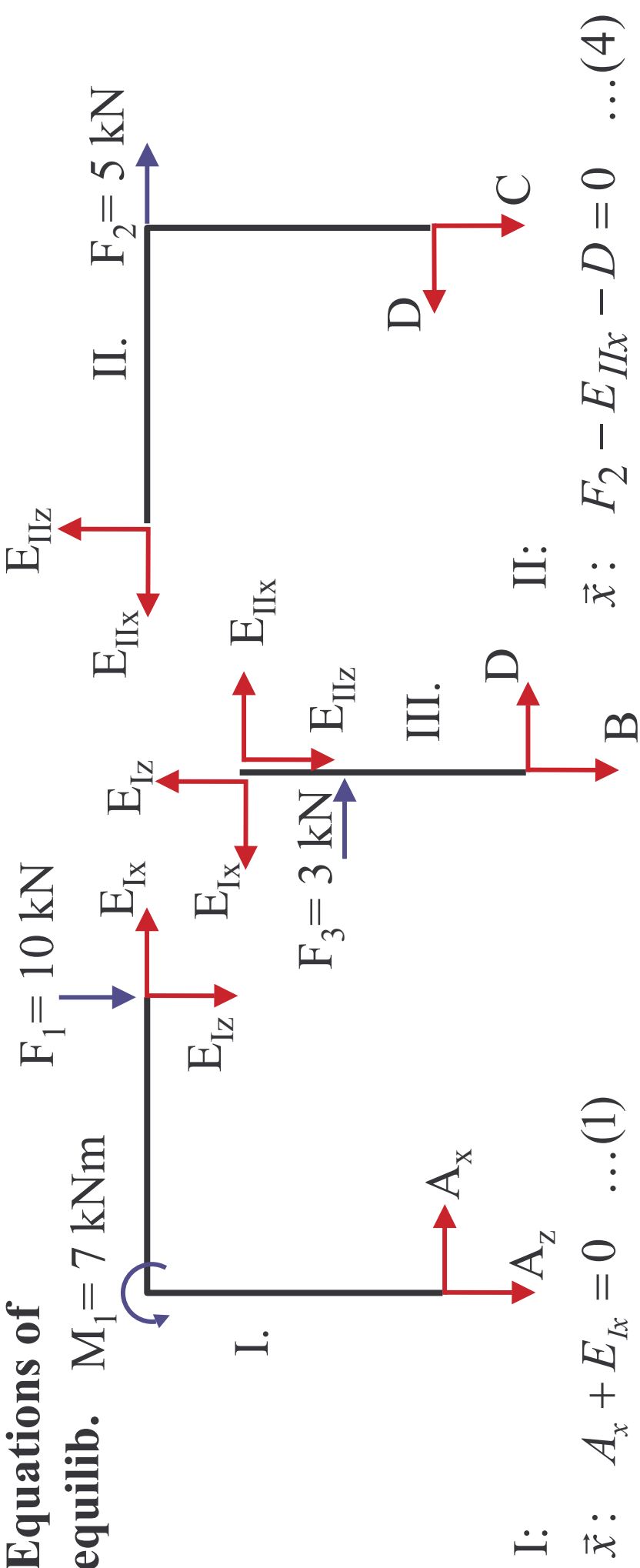
$$m = r$$

# Create free body diagrams

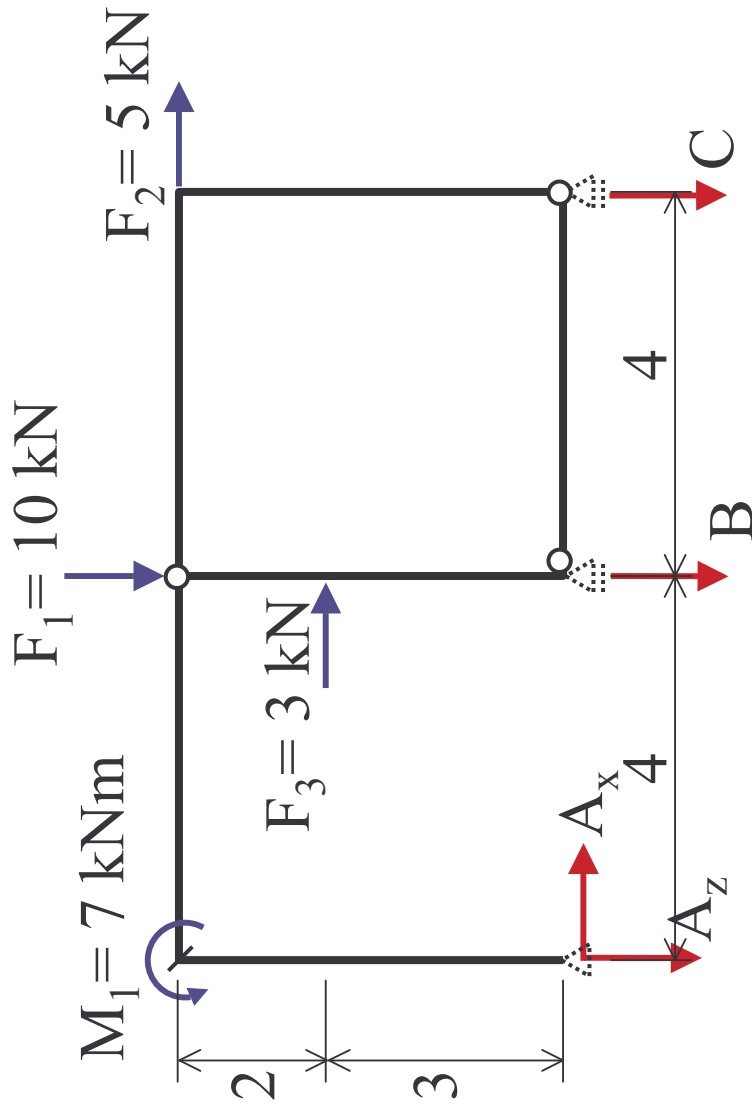


## Equations of

equilib.  $M_1 = 7 \text{ kNm}$



# Equations of equilibrium



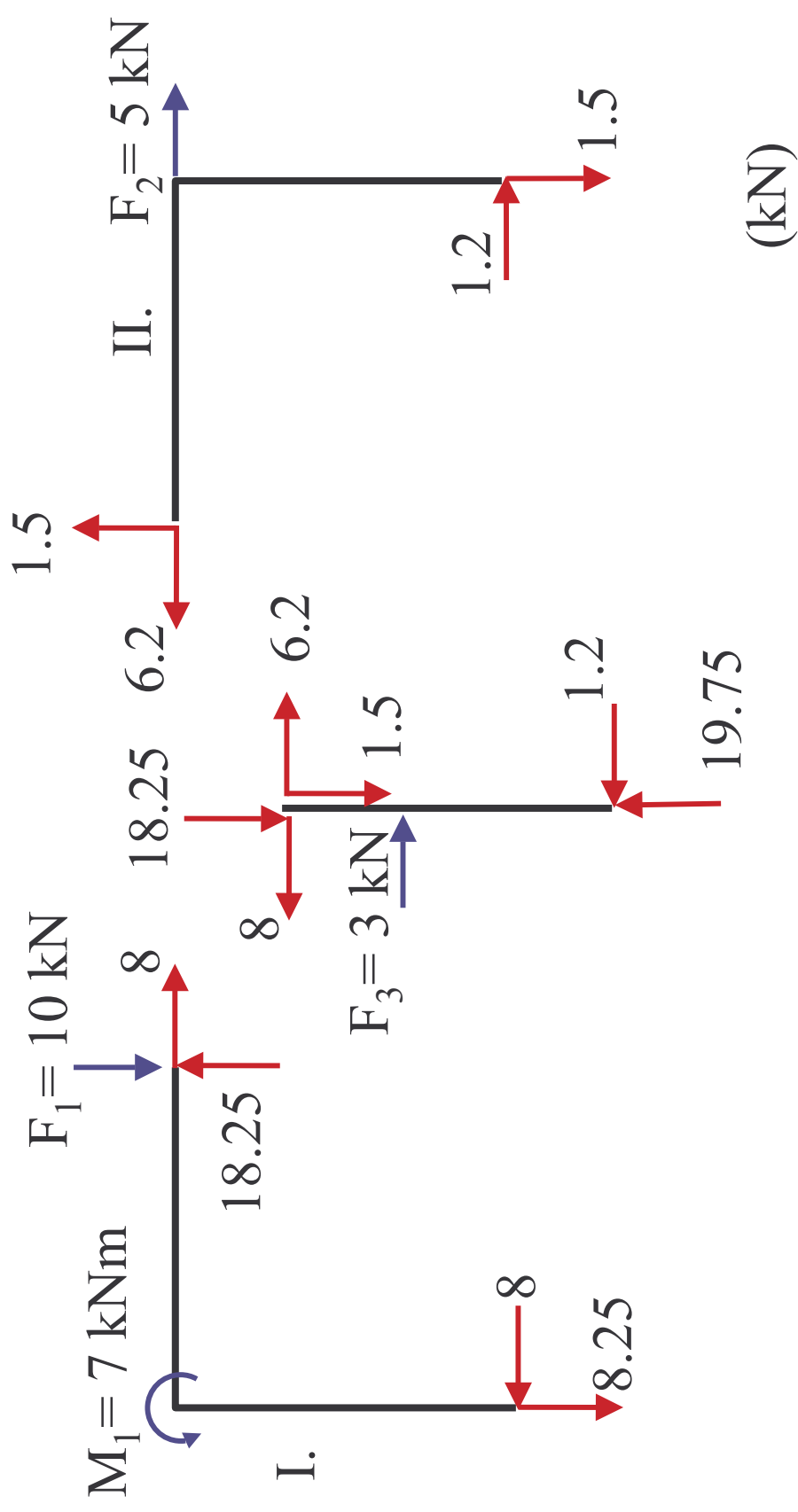
External:

$$\vec{x}: A_x + F_2 + F_3 = 0 \quad \dots(10)$$

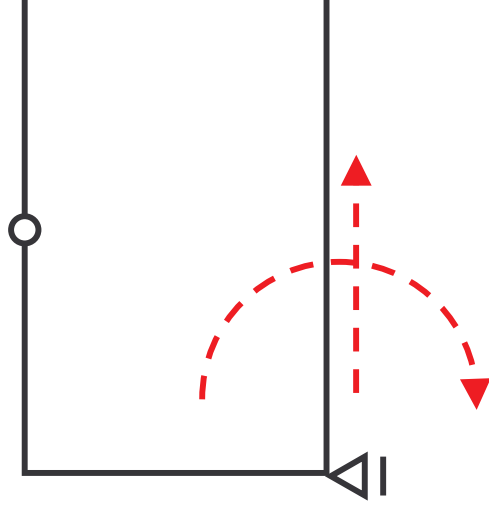
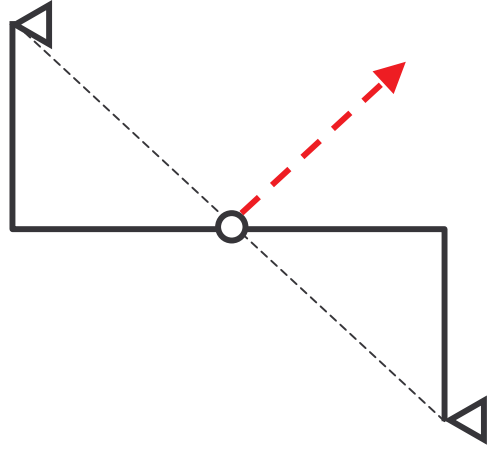
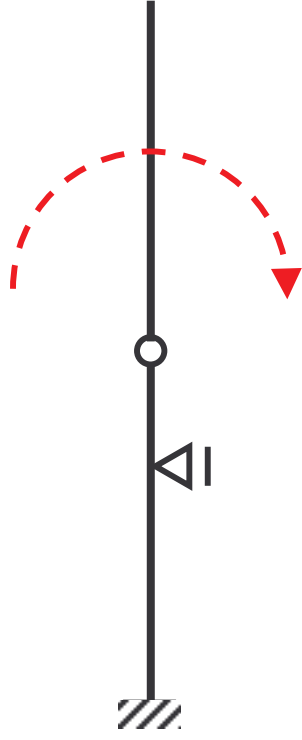
$$\downarrow z: A_z + B + C + F_1 = 0 \quad \dots(11)$$

$$\vec{a}: M_1 - 4F_1 - 5F_2 - 3F_3 - 4B - 8C = 0 \quad \dots(9)$$

**Result:**



\* Improper supports, e.g.:



*Acknowledgment:*  
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*appear courtesy of National Information Service for Earthquake Engineering,*  
*University of California, Berkeley.*