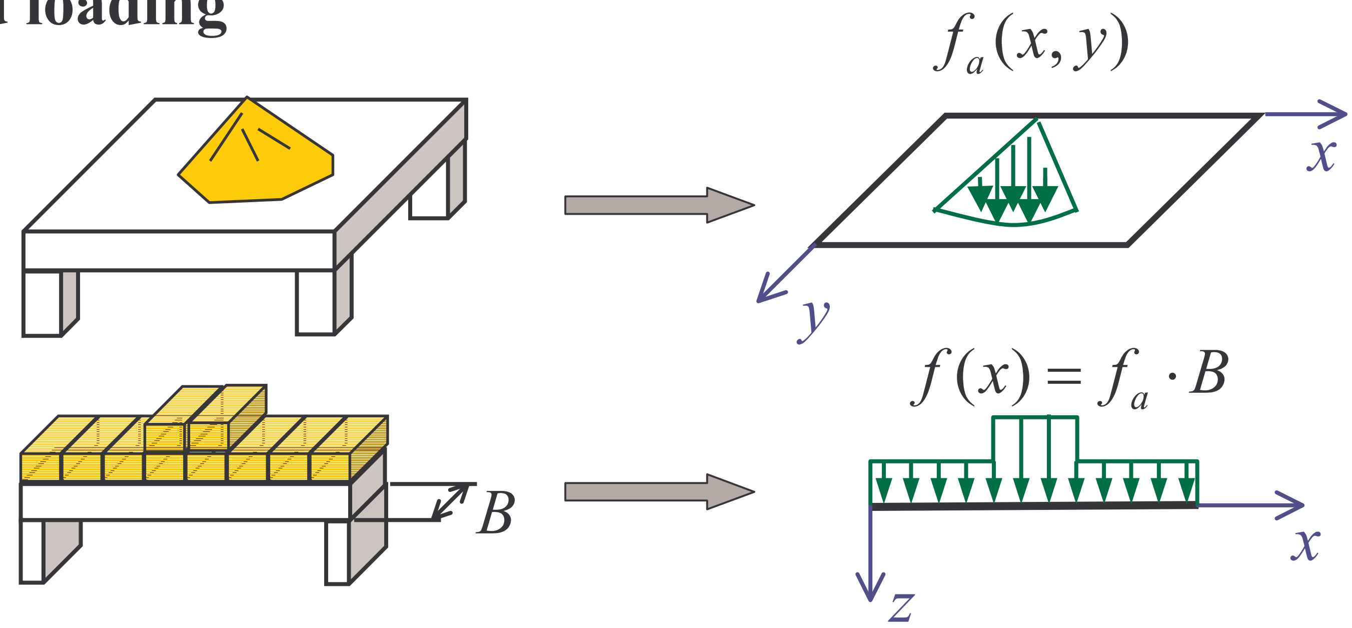
4.5.2 Distributed loading

Plane (2D)
(N/m²)

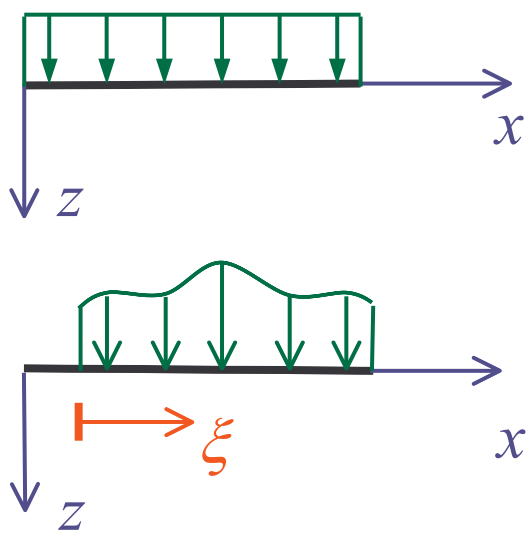
Line (1D)
(N/m)



Distributed line loading

uniform $f = konst.$

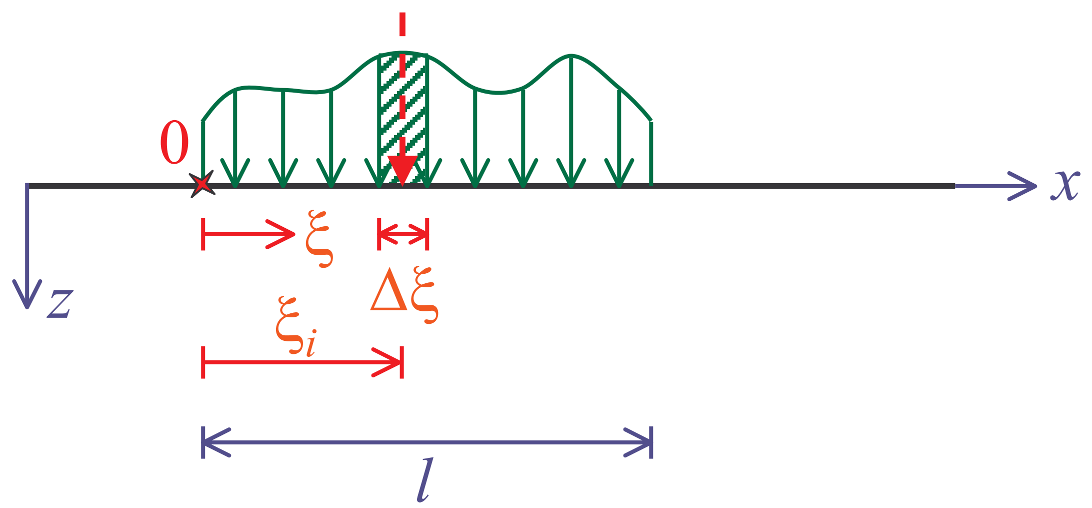
variable $f = f(\xi)$



Resultant of the distribute loading

(substitute force)

$$\Delta F_i = f(\xi_i)\cdot\Delta\xi$$



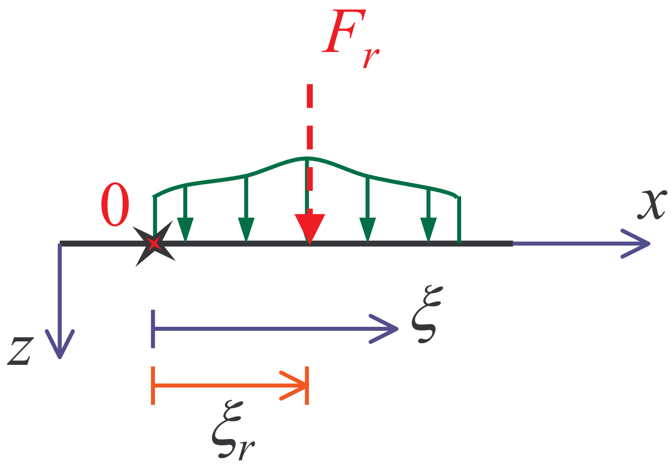
Resultant force

$$F_r = \sum_i \Delta F_i = \sum_i f(\xi_i)\cdot\Delta\xi \qquad \overset{\text{Limit}}{\Delta\xi \to 0} \qquad F_r = \int_0^l f(\xi)\, d\xi$$

Resultant static moment w.r.t. point 0

$$M_0 = -\sum_i \Delta F_i \cdot \xi_i = -\sum_i f(\xi_i)\cdot \xi_i \cdot \Delta\xi \qquad \overset{\text{Limit}}{\Delta\xi \to 0} \qquad M_0 = -\int_0^l f(\xi)\cdot\xi\, d\xi$$

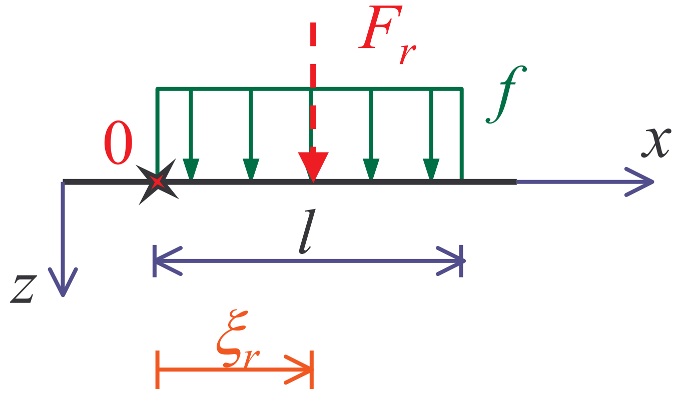
Point of application of the substitute force



- from the moment equivalency condition

$$M_0 = -F_r \cdot \xi_r \quad \Rightarrow \quad \xi_r = \frac{\int_0^l f(\xi) \cdot \xi d\xi}{F_r}$$

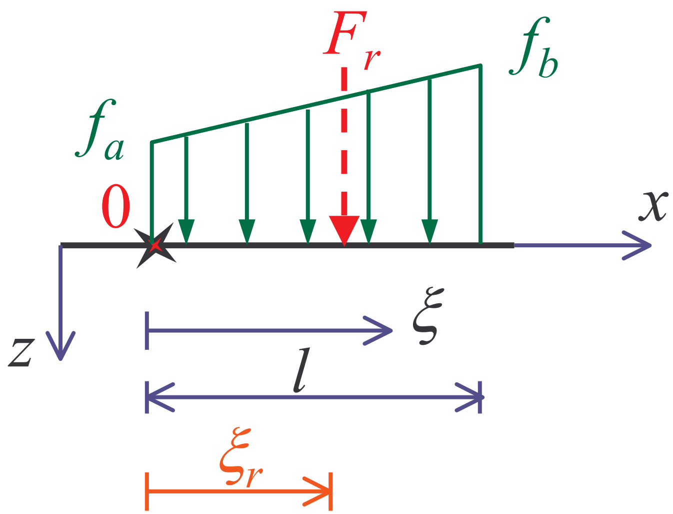
- uniform



$$f(\xi) = f$$

$$F_r = f \cdot l \qquad \xi_r = \frac{l}{2}$$

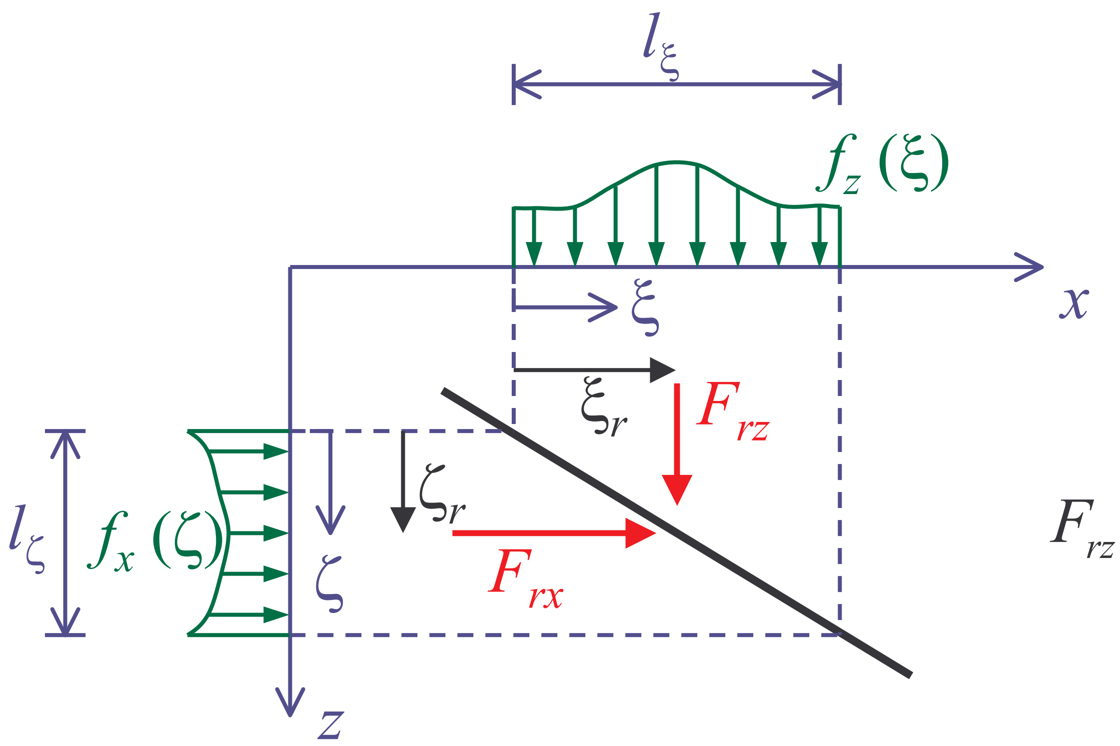
- linear



$$f(\xi) = f_a + \frac{f_b - f_a}{l}\xi$$

$$F_r = \frac{f_a + f_b}{2} \cdot l \qquad \xi_r = \frac{l}{3} \cdot \frac{f_a + 2f_b}{f_a + f_b}$$

General components in directions x and z

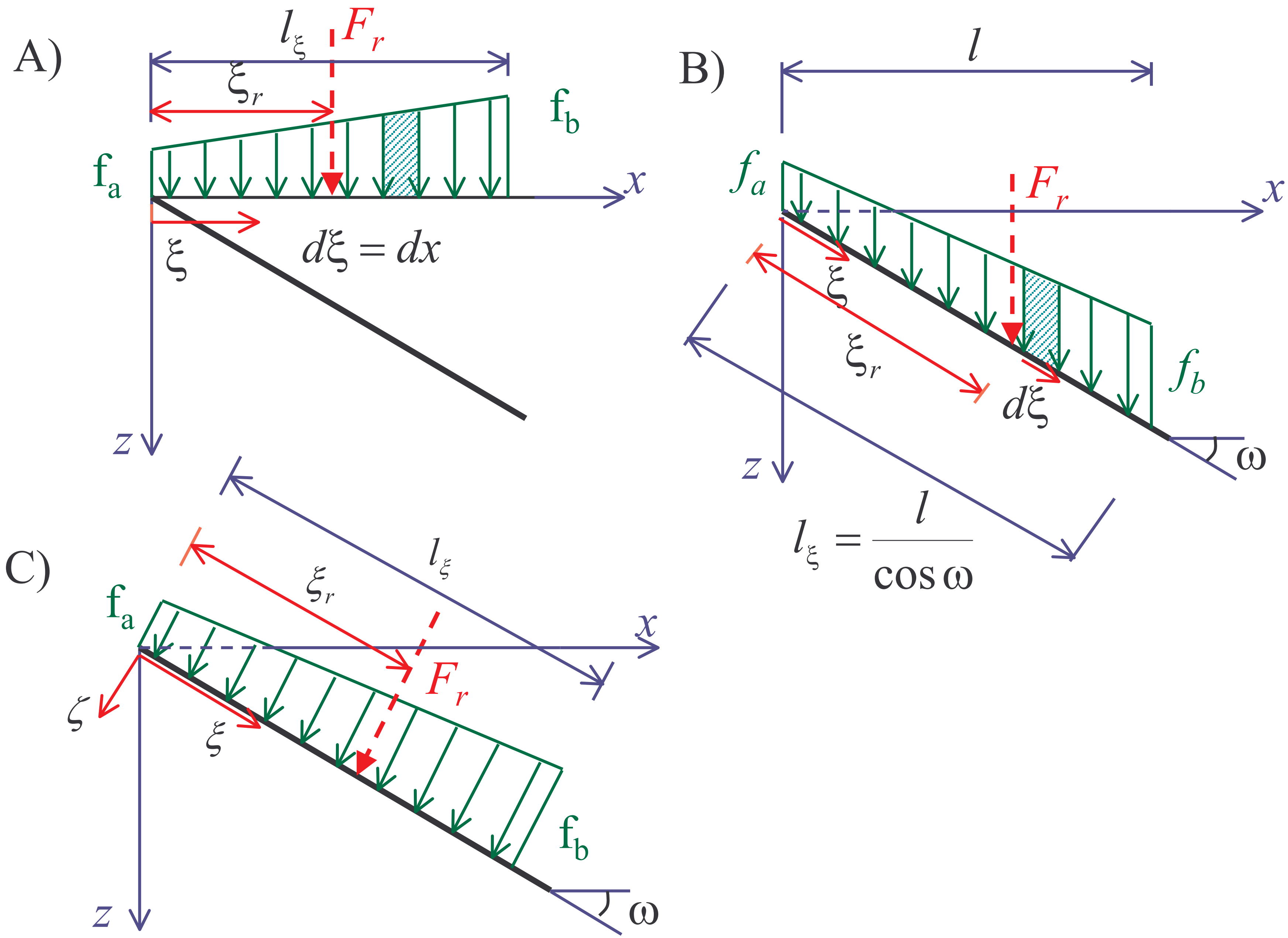


$$F_{rz} = \int_0^{l_\xi} f_z(\xi)\, d\xi$$

$$\xi_r = \frac{\int_0^{l_\xi} f_z(\xi)\cdot \xi\, d\xi}{F_{rz}}$$

$$F_{rx} = \int_0^{l_\zeta} f_x(\zeta)\, d\zeta$$

$$\zeta_r = \frac{\int_0^{l_\zeta} f_x(\zeta)\cdot \zeta\, d\zeta}{F_{rx}}$$

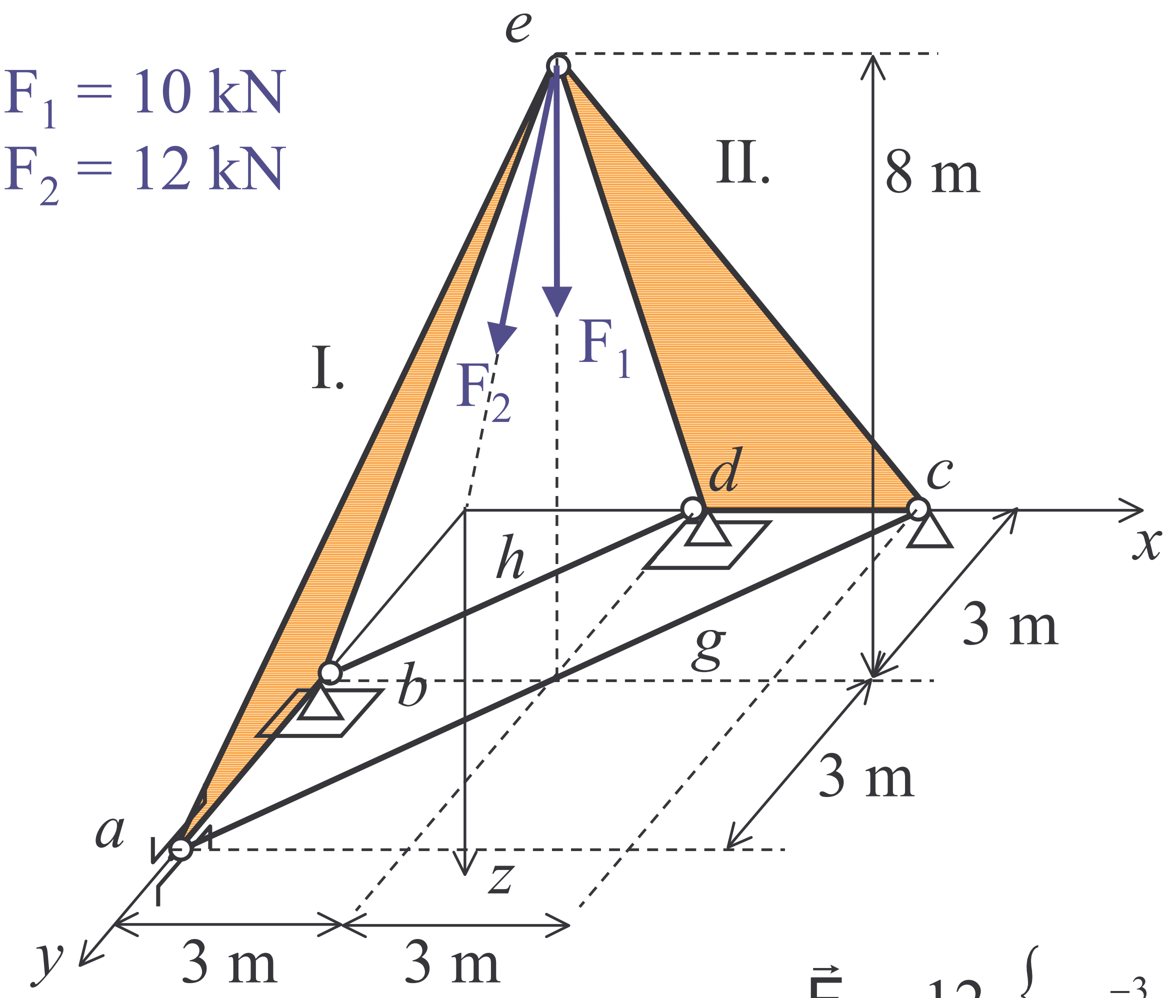


A)
l_ξ
F_r
ξ_r
f_b
f_a
x
ξ
$d\xi = dx$
z
B)
l
f_a
F_r
x
ξ
ξ_r
f_b
$d\xi$
ω
z
$l_\xi = \frac{l}{\cos\omega}$
C)
l_ξ
ξ_r
f_a
x
F_r
ζ
ξ
f_b
z
ω

4.5.3 3D (Space) composed rigid bodies

Ex.: Determine reactions of the system of rigid bodies

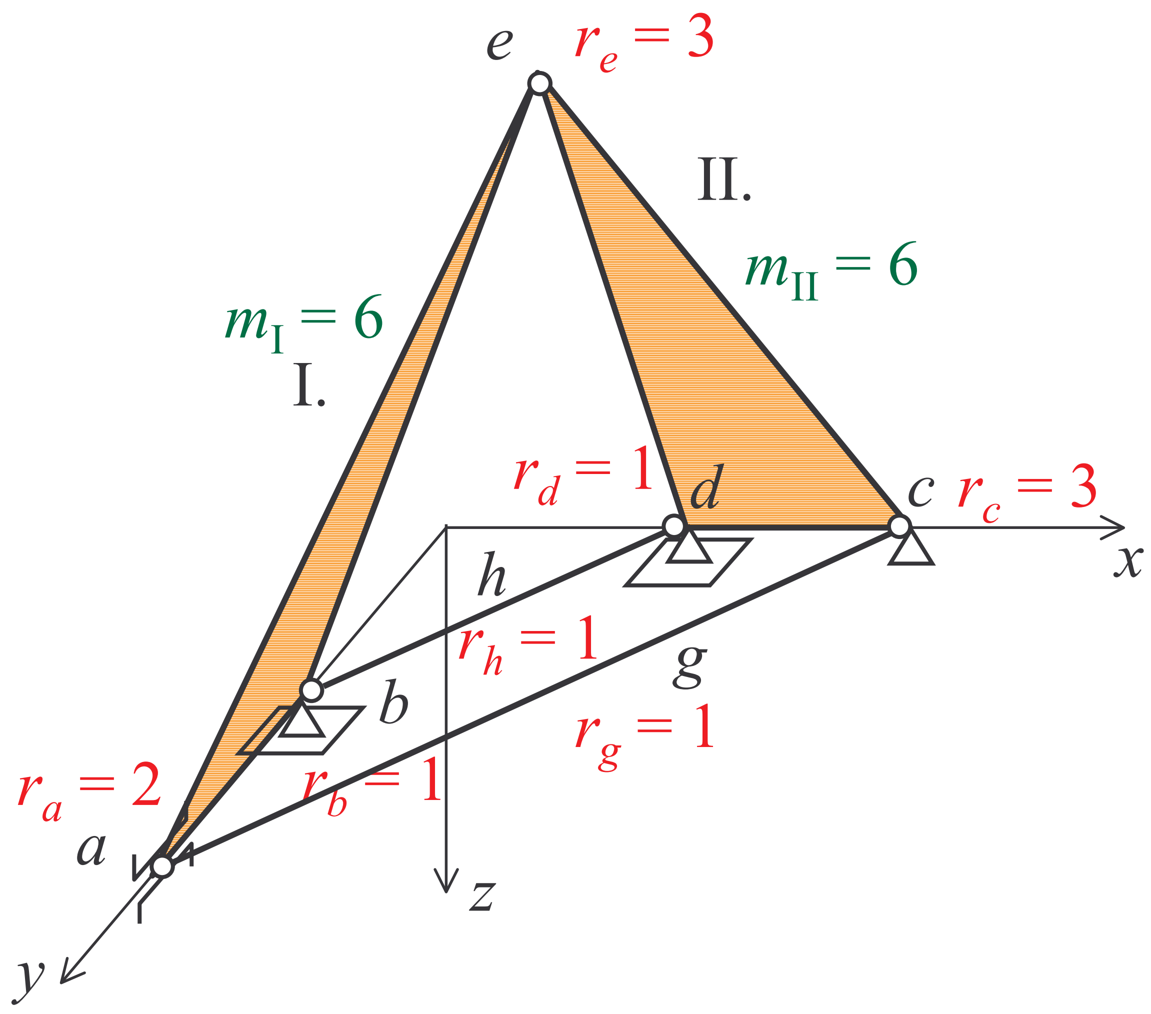
$F_1 = 10$ kN
$F_2 = 12$ kN



Load vectors:

$$\vec{F}_1 = \{0,\ 0,\ 10\}\ kN$$

$$\vec{F}_2 = 12 \cdot \left\{ \frac{-3}{\sqrt{3^2+3^2+8^2}},\ \frac{-3}{\sqrt{3^2+3^2+8^2}},\ \frac{8}{\sqrt{3^2+3^2+8^2}} \right\}$$
$$= \{-3.97553,\ -3.97553,\ 10.6014\}\quad kN$$



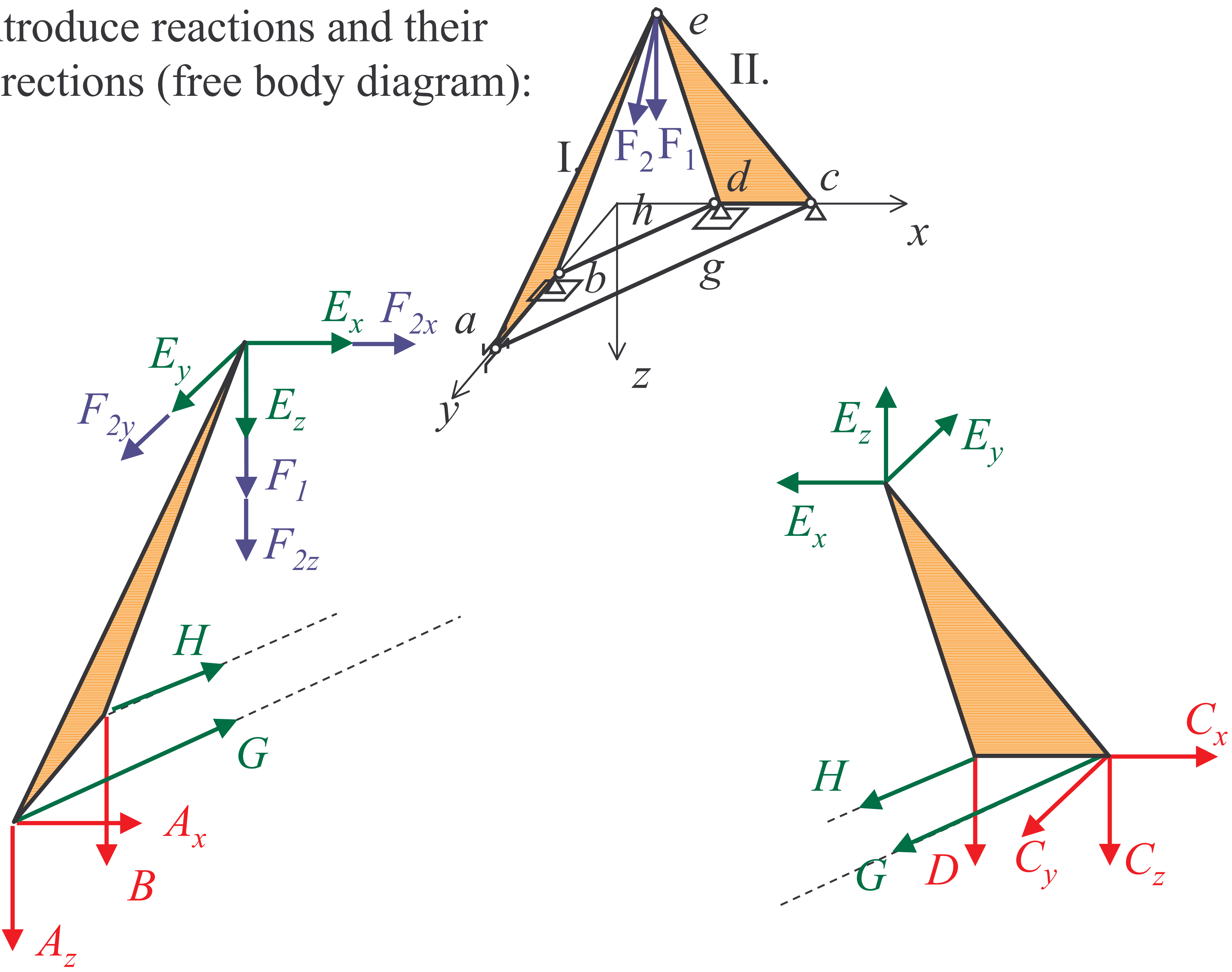
Determine s:

$m = 6 + 6 = 12$

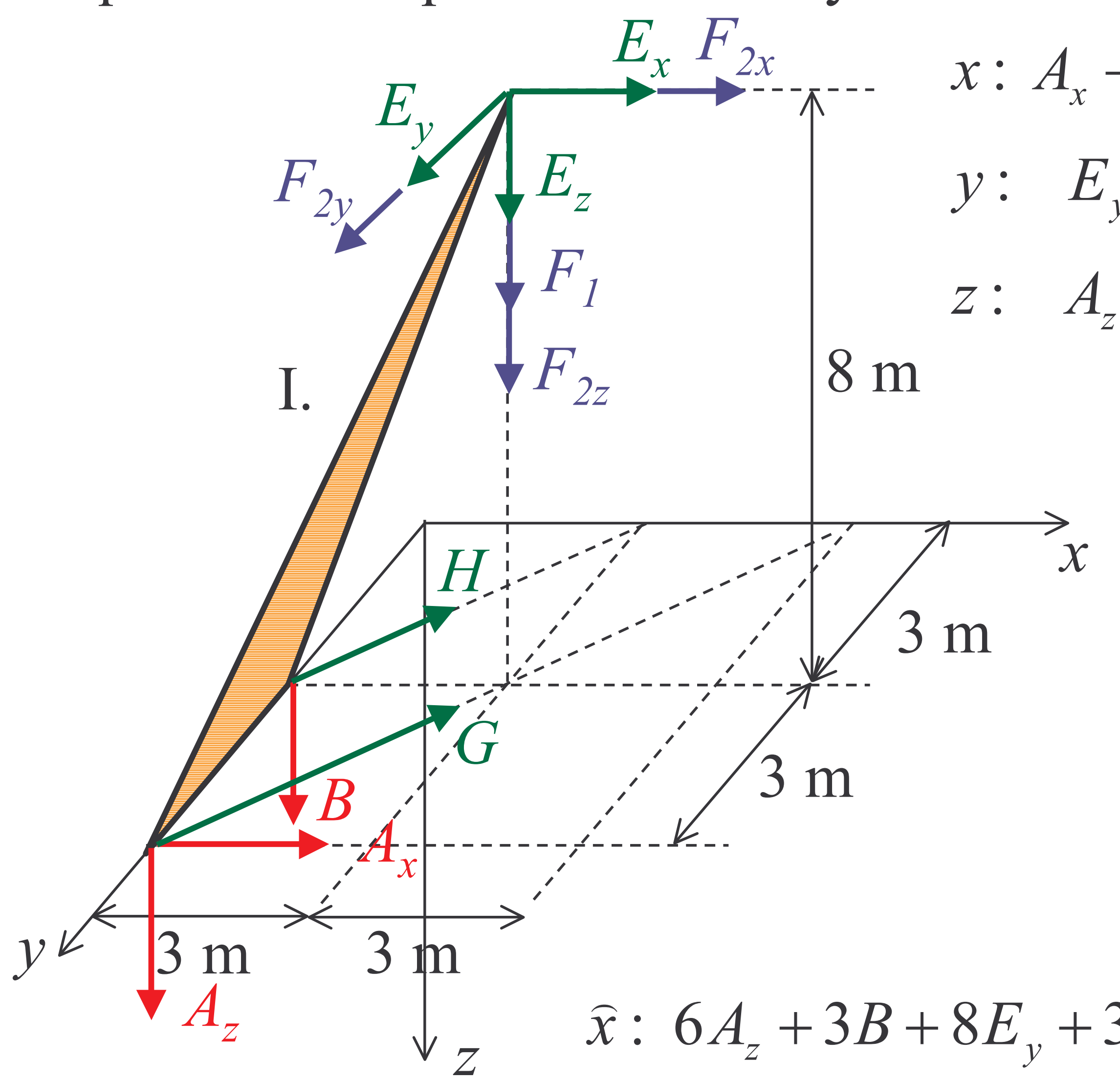
$r = 4\times1 + 2 + 2\times3 = 12$

$m = r$ ...stat. determinate
if not an improperly
supported structure.

Introduce reactions and their directions (free body diagram):



Equations of equilibrium – body #I.:



$$x: \; A_x + E_x + G\tfrac{\sqrt{2}}{2} + H\tfrac{\sqrt{2}}{2} + F_{2x} = 0 \quad (1)$$

$$y: \; E_y - G\tfrac{\sqrt{2}}{2} - H\tfrac{\sqrt{2}}{2} + F_{2y} = 0 \quad (2)$$

$$z: \; A_z + B + E_z + F_1 + F_{2z} = 0 \quad (3)$$

$$\hat{x}: \; 6A_z + 3B + 8E_y + 3E_z + 3F_1 + 8F_{2y} + 3F_{2z} = 0 \quad (4)$$

$$\hat{y}: \; -8E_x - 3E_z - 3F_1 - 8F_{2x} - 3F_{2z} = 0 \quad (5)$$

$$\hat{z}: \; -6A_x - 3E_x + 3E_y - 6G\tfrac{\sqrt{2}}{2} - 3H\tfrac{\sqrt{2}}{2} - 3F_{2x} + 3F_{2y} = 0 \quad (6)$$

Equations of equilibrium – body #II.:

$$x: \quad C_x - E_x - G\tfrac{\sqrt{2}}{2} - H\tfrac{\sqrt{2}}{2} = 0 \qquad (7)$$

$$y: \quad C_y - E_y + G\tfrac{\sqrt{2}}{2} + H\tfrac{\sqrt{2}}{2} = 0 \qquad (8)$$

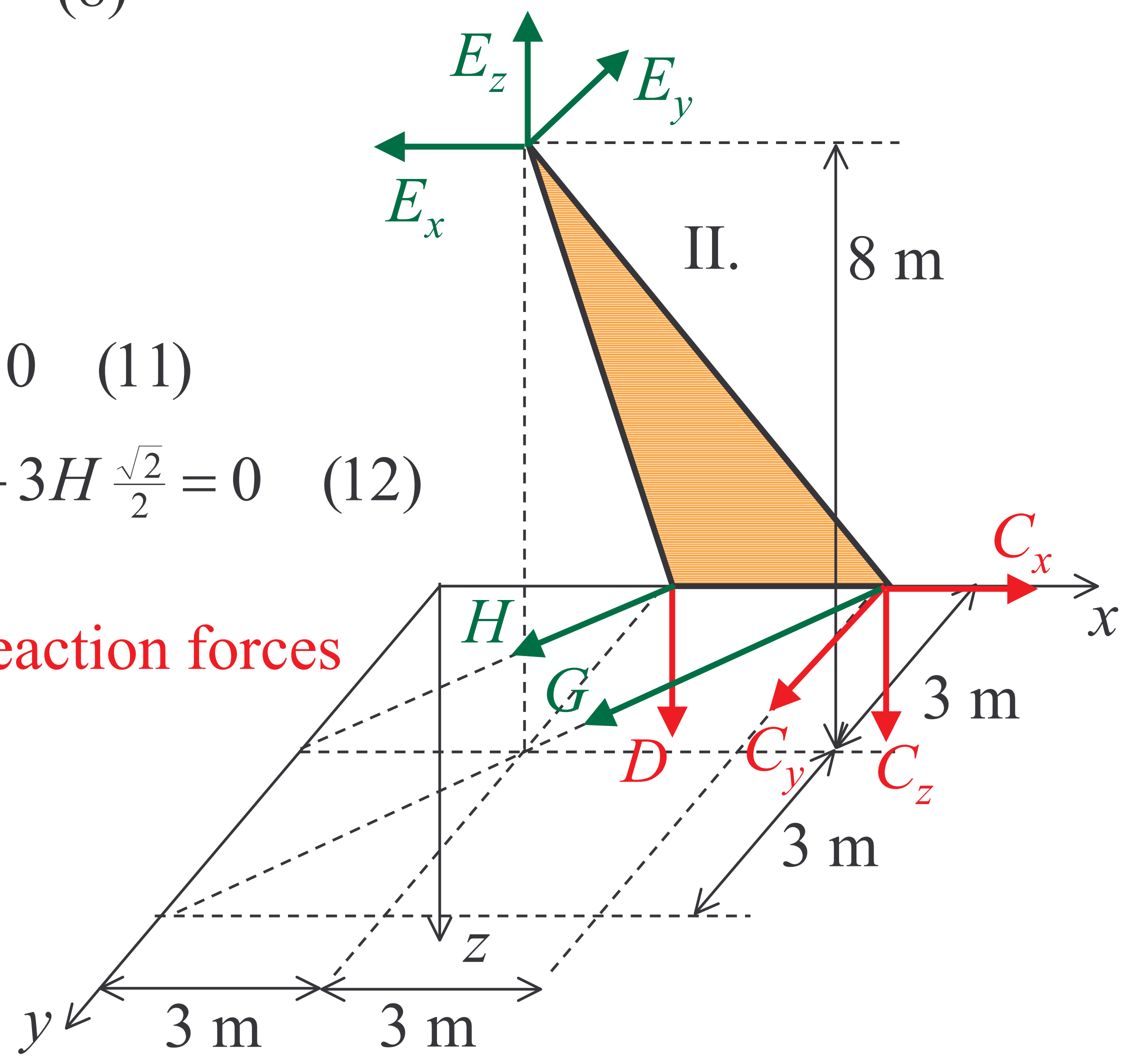
$$z: \quad C_z + D - E_z = 0 \qquad (9)$$

$$\hat{x}: \quad -8E_y - 3E_z = 0 \qquad (10)$$

$$\hat{y}: \quad -6C_z - 3D + 8E_x + 3E_z = 0 \qquad (11)$$

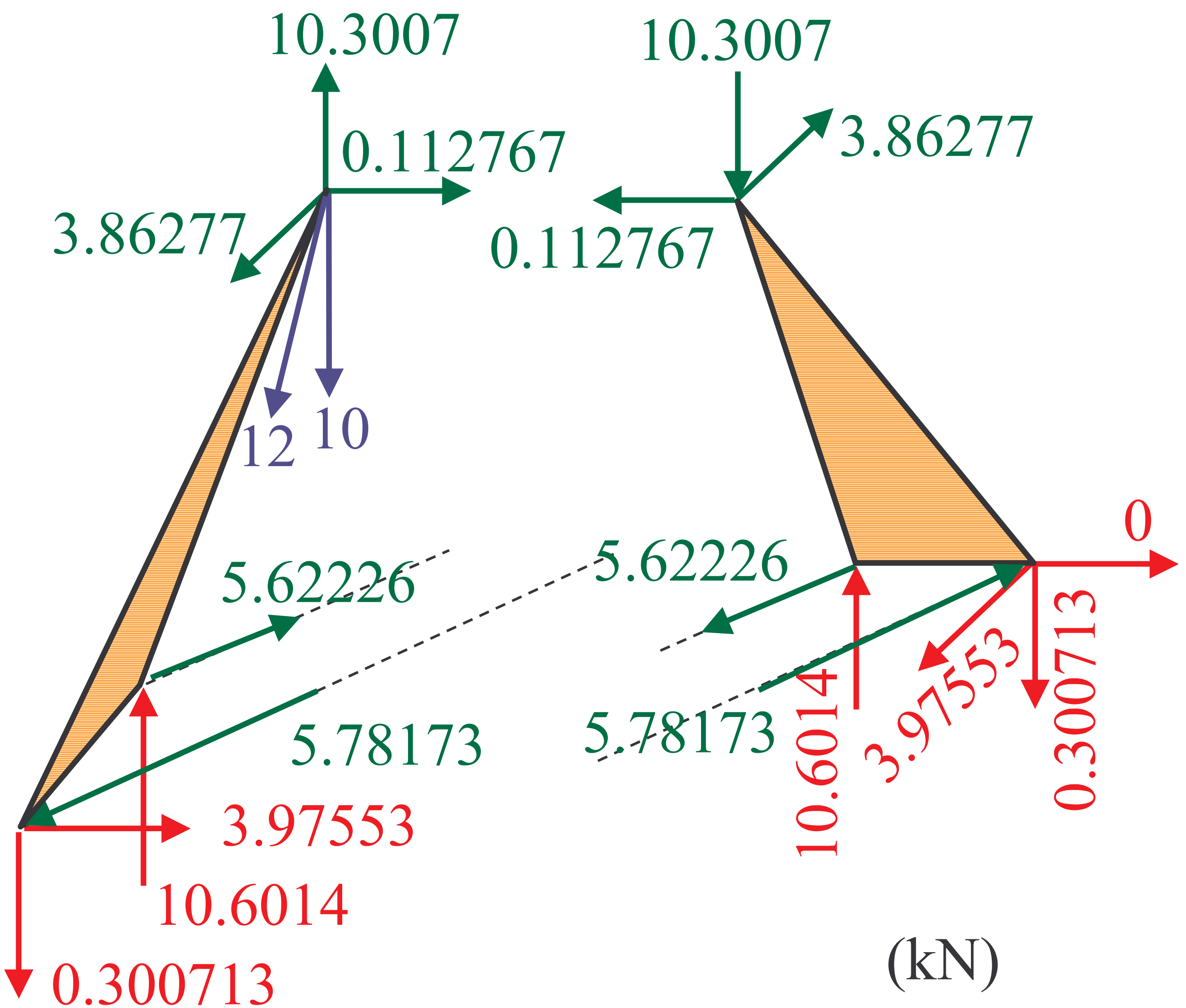
$$\hat{z}: \quad 6C_y + 3E_x - 3E_y + 6G\tfrac{\sqrt{2}}{2} + 3H\tfrac{\sqrt{2}}{2} = 0 \qquad (12)$$

12 equations 12 unknown reaction forces

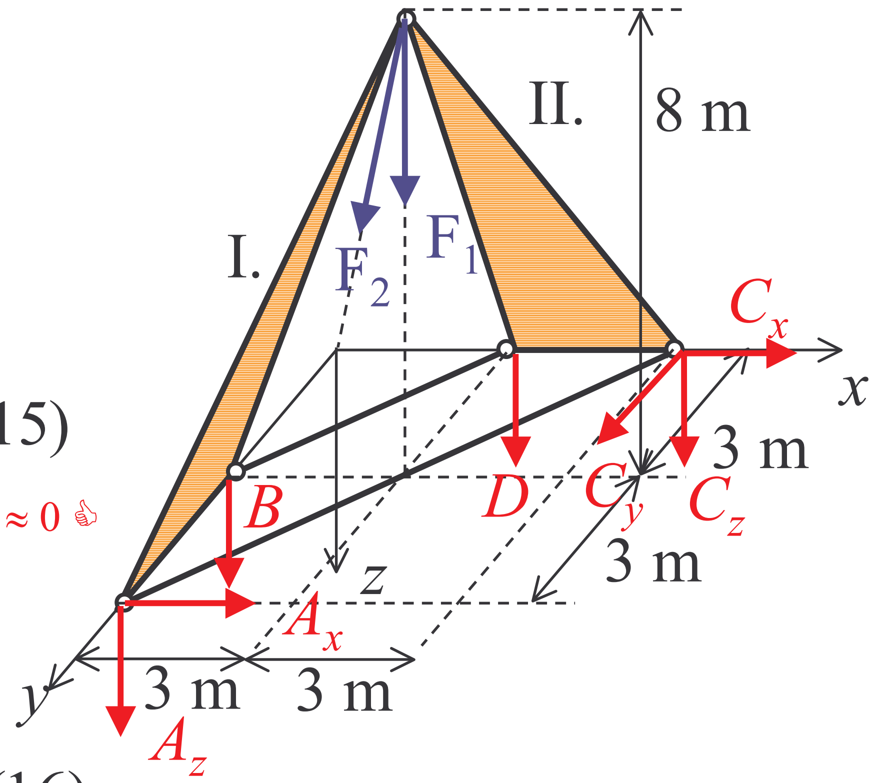


Solution:

$A_x = 3.97553$ kN
$A_z = 0.300713$ kN
$B = -10.6014$ kN
$C_x = 0$ kN
$C_y = 3.97553$ kN
$C_z = 0.300713$ kN
$D = -10.6014$ kN
$E_x = 0.112767$ kN
$E_y = 3.86277$ kN
$E_z = -10.3007$ kN
$G = -5.78173$ kN
$H = 5.62226$



Check – external equations of equilibrium:



$$x: \; A_x + C_x + F_{2x} = 0 \quad (13)$$

3.97553 + 0 - 3.97553 = 0

$$y: \quad C_y + F_{2y} = 0 \quad (14)$$

3.97553 - 3.97553 = 0

$$z: \quad A_z + B + C_z + D + F_1 + F_{2z} = 0 \quad (15)$$

0.300713 - 10.6014 + 0.300713 - 10.6014 + 10 + 10.6014 ≈ 0

$$\hat{x}: \; 6A_z + 3B + 3F_1 + 8F_{2y} + 3F_{2z} = 0 \quad (16)$$

6 • 0.300713 - 3 • 10.6014 + 3 • 10 - 8 • 3.97553 + 3 • 10.6014 ≈ 0

$$\hat{y}: \quad -6C_z - 3D - 3F_1 - 8F_{2x} - 3F_{2z} = 0 \quad (17)$$

-6 • 0.300713 + 3 • 10.6014 - 3 • 10 + 8 • 3.97553 - 3 • 10.6014 ≈ 0

$$\hat{z}: \; -6A_x + 6C_y - 3F_{2x} + 3F_{2y} = 0 \quad (18)$$

-6 • 3.97553 + 6 • 3.97553 + 3 • 3.97553 - 3 • 3.97553 = 0