Engineering mechanics 10 (K132EM10)

Lecturer:

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Literature:

Kufner, Kuklík: Stavební mechanika 10, ES ČVUT
Kufner, Kuklík: Stavební mechanika 20, ES ČVUT
Lecture notes

Course requirements (max. 100 points)

* 2 tests (17 + 17 = 34 points):

  - minimum 50% (t.j. 17 points)
  - only one substitute test will be opened for those not able to come at regular time

* homework:

  - 100% correctly solved and submitted in due time

Exam (max. 66 points)

* Written part

  - 3-4 solution problems
  - 3-4 theoretical questions

* Oral part - to adjust the final evaluation if necessary

Final evaluation

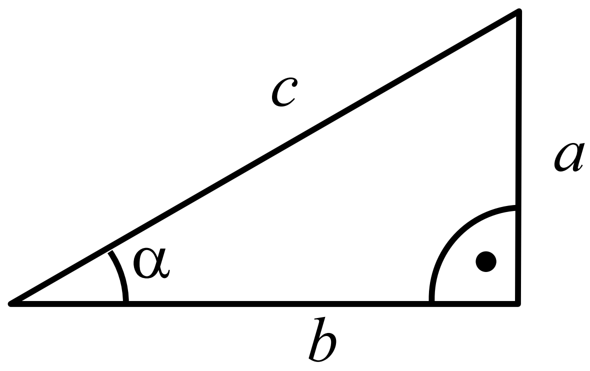
points	mark
100 - 86	A
85 - 70	B
69 - 50	C
49 - 0	D

2. Review of some basic relations

2.1 Trigonometry

Rectangular triangle

$\sin\alpha = a/c$
$\cos\alpha = b/c$
$\tan\alpha = a/b$



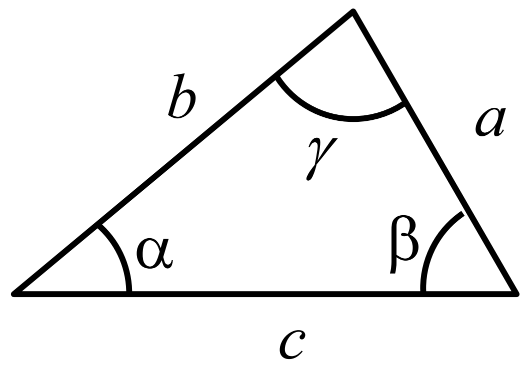
General triangle

Sine law:

$$a : b : c = \sin\alpha : \sin\beta : \sin\gamma$$

Cosine law:

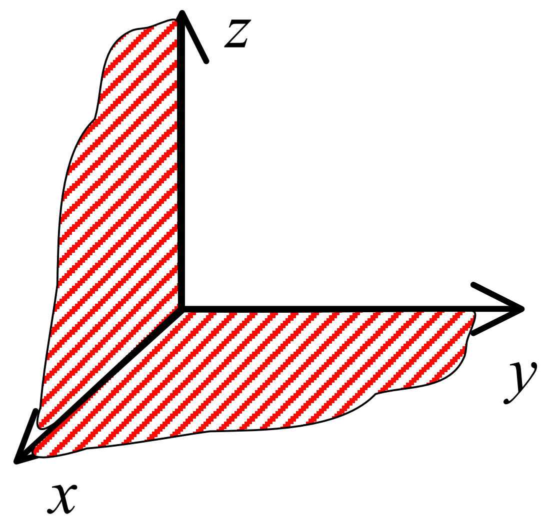
$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$
$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$



2. Review of some basic relations

2.2 Vector algebra

2.2.1 Cartesian coordinate system



Coordinate system is space:

- system of three perpendicular axis x, y, z
- right-handed coordinate system: rotation

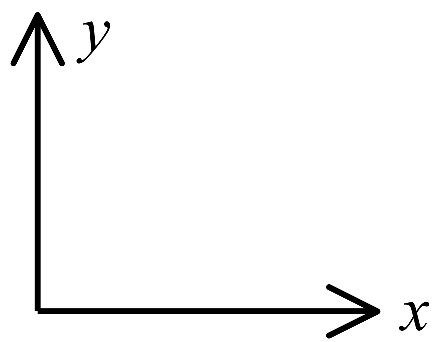
  x $\rightarrow$ y in positive sense about axis z

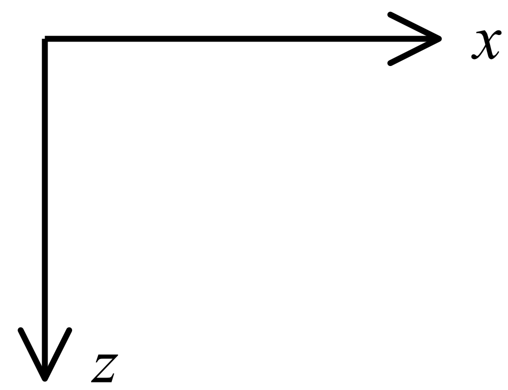
  y $\rightarrow$ z in positive sense about axis x

  z $\rightarrow$ x in positive sense about axis y

  (positive sense – counterclockwise)

Coordinate system in plane:





2.2.2 Vector

Scalar:

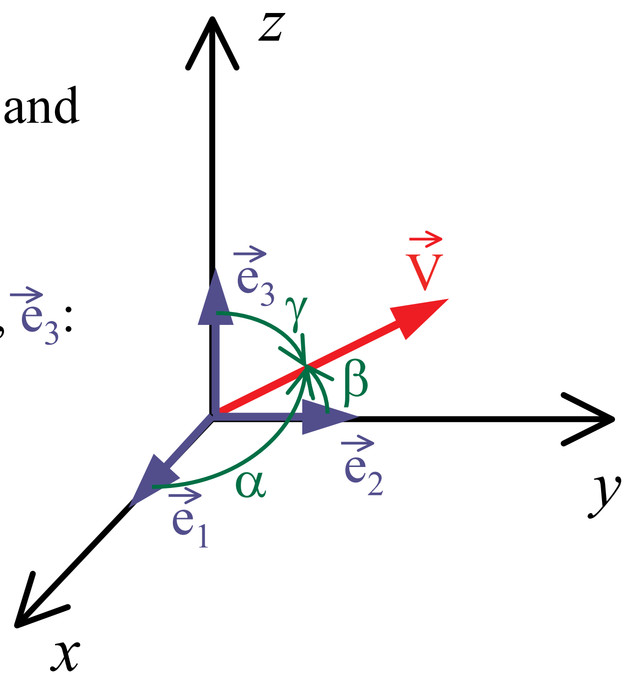
- a quantity having only magnitude, does not depend on the selection of the coordinate system

Vector $\vec{V}$:

- a quantity specified by magnitude, direction and orientation
- always related to a given coordinate system

Base vectors - coordinate (unit) vectors) $\vec{e}_1, \vec{e}_2, \vec{e}_3$:

- jednotkové vektory v kladných směrech souřadnicových os



Direction angles α, β, γ:

- angles determined by vector $\vec{V}$ and positive coordinate axis
- it holds $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Rectangular representation of vector using <u>components</u>:

- components: perpendicular projections of vector into direction of individual coordinate axis
- $\vec{V} = \{ V_x; V_y; V_z \}$
- using directional angles:

  $V_x = V \cos\alpha$

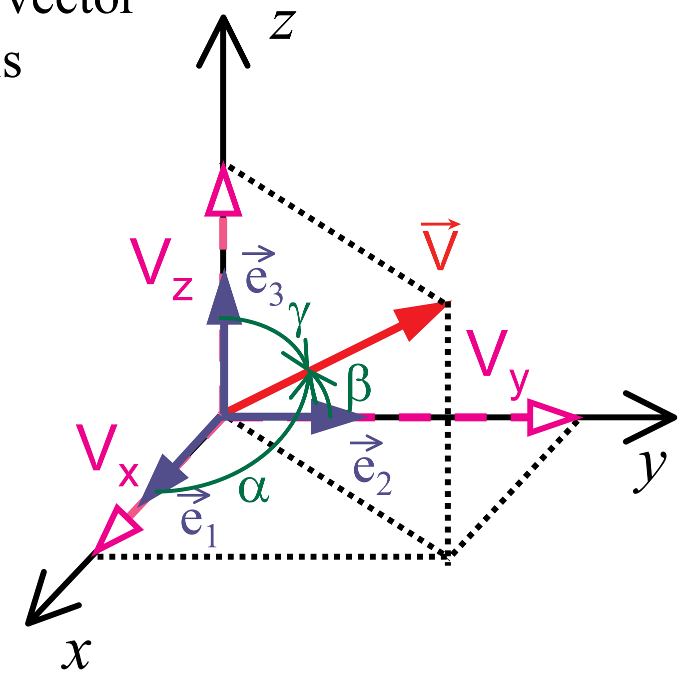
  $V_y = V \cos\beta$

  $V_z = V \cos\gamma$
- V ... magnitude of vector $\vec{V}$:

  $V = |\vec{V}| = (V_x^2 + V_y^2 + V_z^2)^{1/2}$
- base (unit) vectors: $\vec{e}_1 = \{1; 0; 0\}$

  $\vec{e}_2 = \{0; 1; 0\}$

  $\vec{e}_3 = \{0; 0; 1\}$

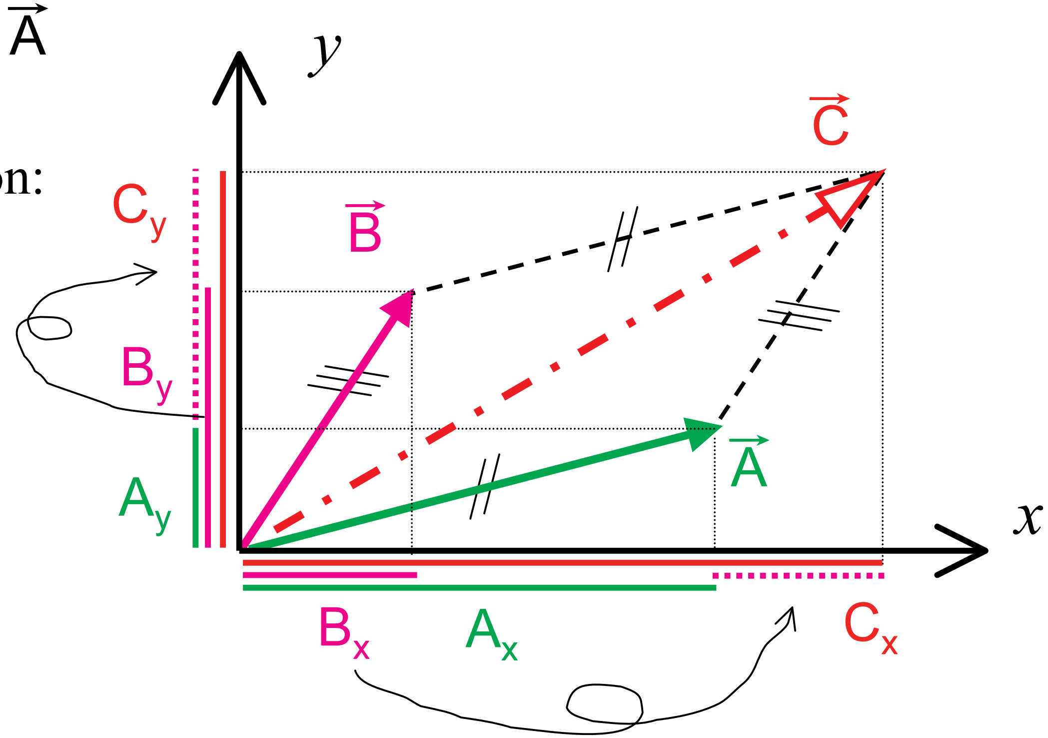


2.2.3 Vector algebra

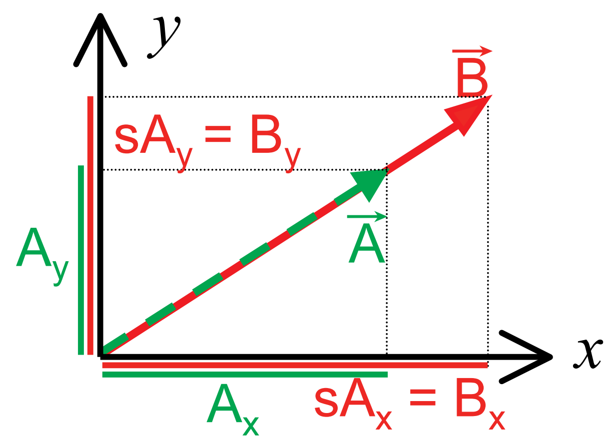
<u>Sum</u> of vectors $\vec{A}$ and $\vec{B}$ results in a vector $\vec{C}$ given by:

$$\vec{C} = \{ A_x+B_x;\ A_y+B_y;\ A_z+B_z \}$$

- notation: $\vec{C} = \vec{A} + \vec{B}$
- properties: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$
- geometrical representation:



Multiplication of a scalar s and a vector $\vec{A}$ is a vector $\vec{B}$, given by:



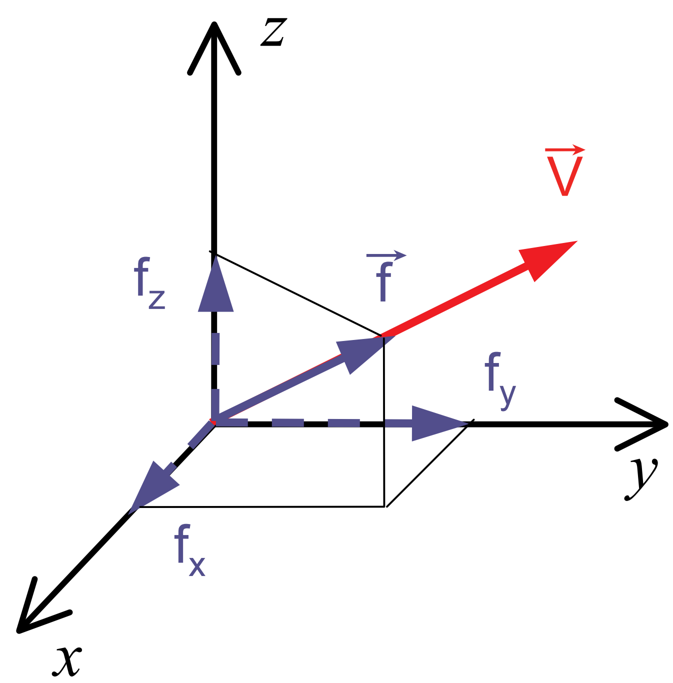
$$\vec{B} = \{s\,A_x, s\,A_y, s\,A_z\}$$

- notation: $\vec{B} = s\,\vec{A}$
- properties:
  - * $s\,\vec{A} = \vec{A}\,s$
  - * vectors $\vec{A}$ $\vec{B}$ are parallel
  - * magnitude $B = (s^2A_x{}^2 + s^2A_y{}^2 + s^2A_z{}^2)^{1/2} = s\,A$

Application:

Representation of vector components using components of a unit vector in the direction of $\vec{V}$:

$$\vec{f} = \{ f_x; f_y; f_z \}; \quad |\vec{f}| = f = 1$$



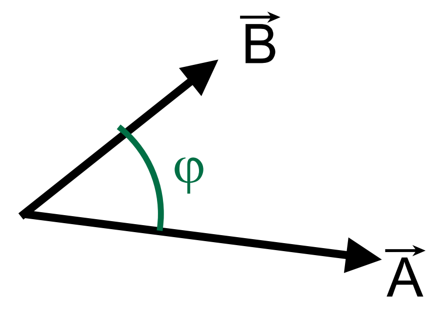
$$V_x = V f_x$$
$$V_y = V f_y$$
$$V_z = V f_z$$

Also

$$f_x = \cos \alpha$$
$$f_y = \cos \beta$$
$$f_z = \cos \gamma$$

<u>Scalar product</u> of two vectors $\vec{A}$ and $\vec{B}$ is a scalar s given by:

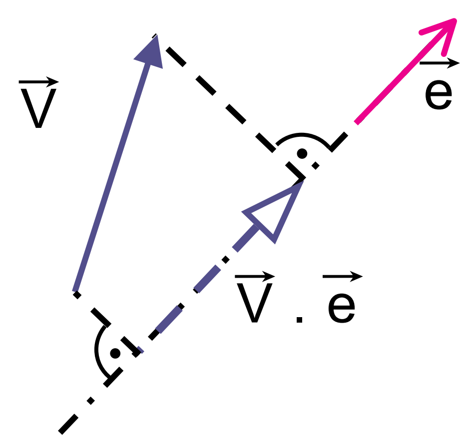
$$s = A\,B\cos\varphi$$
$$= A_x B_x + A_y B_y + A_z B_z$$



- notation: $s = \vec{A} \cdot \vec{B}$
- properties:
    * $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$
    * pro $A \perp B$: $\cos\varphi = 0$, $s = 0$

- geometrical representation and notation:
    * e.g., representation of components of vector $\vec{V}$

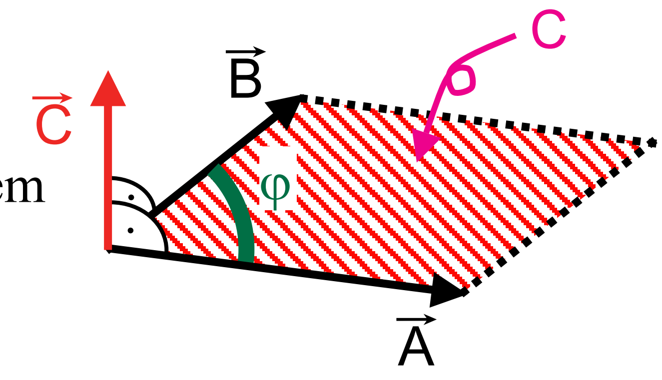
$$V_x = V\cos\alpha = \vec{V} \cdot \vec{e}_1$$
$$V_y = V\cos\beta = \vec{V} \cdot \vec{e}_2$$
$$V_z = V\cos\gamma = \vec{V} \cdot \vec{e}_3$$



* dot product $\vec{V} \cdot \vec{e}$ represents a projection of vector $\vec{V}$ along the axis specified by a unit vector $\vec{e}$

<u>Vector product of two vectors</u> $\vec{A}$ and $\vec{B}$ is a vector $\vec{C}$ having the following properties:

1. magnitude $C = A\,B\,\sin\varphi$ (area of an rectangle)
2. vector $\vec{C}$ is perpendicular to $\vec{A}$ and $\vec{B}$
3. vectors $\vec{A}, \vec{B}, \vec{C}$ form a right-handed coord. system



- notation: $\vec{C} = \vec{A} \times \vec{B}$
- properties:

  * $\vec{A} \times \vec{B} = -\,\vec{B} \times \vec{A}$
  * $s\,(\vec{A} \times \vec{B}) = (s\,\vec{A}) \times \vec{B} = \vec{A} \times (s\,\vec{B})$
  * $(\vec{A} + \vec{B}) \times \vec{D} = \vec{A} \times \vec{D} + \vec{B} \times \vec{D}$

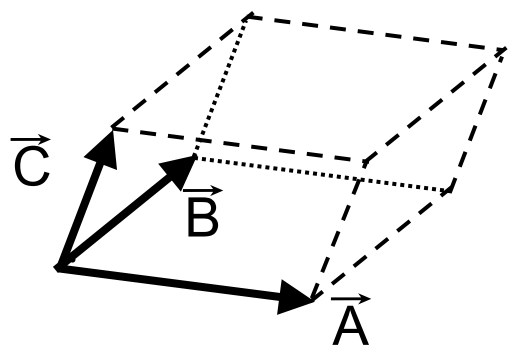
- components

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - B_y A_z)\vec{e}_1 + (A_z B_x - B_z A_x)\vec{e}_2 + (A_x B_y - B_x A_y)\vec{e}_3$$

$$= C_x \vec{e}_1 + C_y \vec{e}_2 + C_z \vec{e}_3 = \{C_x, C_y, C_z\}$$

- <u>multiplication of three vectors</u> $\vec{A}$, $\vec{B}$ and $\vec{C}$ is a scalar s defined by determinat:

$$s = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix} = A_xB_yC_z + A_yB_zC_x + A_zB_xC_y - A_zB_yC_x - A_yB_xC_z - A_xB_zC_y$$



- notation: $s = (\vec{A} \times \vec{B}) . \vec{C}$
- geometrical representation: volume of a solid determined by vectors $\vec{A}$, $\vec{B}$, $\vec{C}$
- properties:

  * $(\vec{A} \times \vec{B}) . \vec{C} > 0$ if vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ are not found in a single plane and form a right-handed coordinate system
  * $(\vec{A} \times \vec{B}) . \vec{C} = 0$ if vectors $\vec{A}$, $\vec{B}$, $\vec{C}$ are found in a single plane or if at least one of then is a zero vector

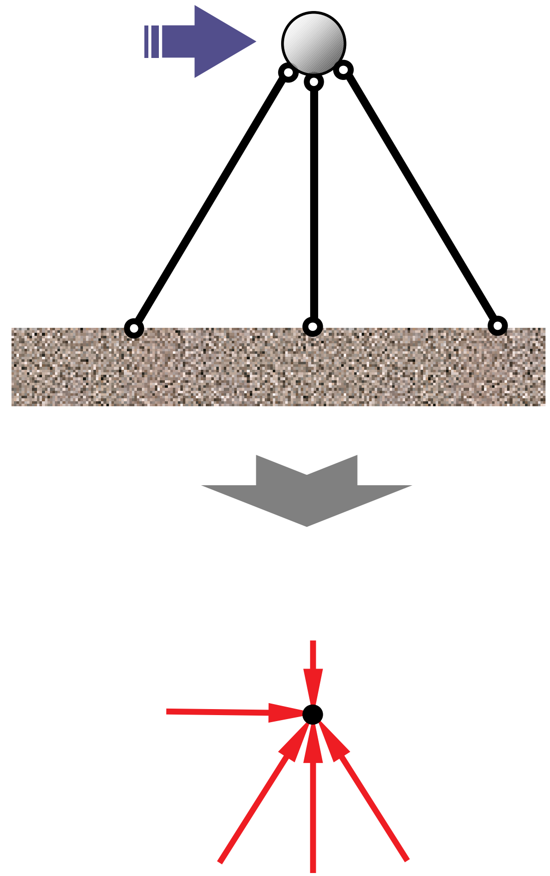
  $(\vec{A} \times \vec{B}) . \vec{C} = \vec{C} . (\vec{A} \times \vec{B})$

  * $(\vec{A} \times \vec{B}) . \vec{C} = -(\vec{B} \times \vec{A}) . \vec{C}$
  * $(\vec{A} \times \vec{B}) . \vec{C} = (\vec{B} \times \vec{C}) . \vec{A} = (\vec{C} \times \vec{A}) . \vec{B}$

3. Geometry of forces

3.1 Concurrent system of forces

3.1.1 Problem assignment, assumptions



Task:
Mathematically describe mechanical effects of loading on a structure and effects of one part of a structure on to the other.

Simplifying assumption:
structure and its parts can be idealized by a point.

Effects will be described by a vector quantity – force.

3.1.2 Force

- notation $\vec{F}$, $\vec{R}$
- definition, e.g., Newton law of force:

  Change of momentum of a mass point with respect to time equals the force acting on this point

$$\frac{d\vec{H}}{dt} = \frac{d(m\vec{v})}{dt} = \vec{F}$$

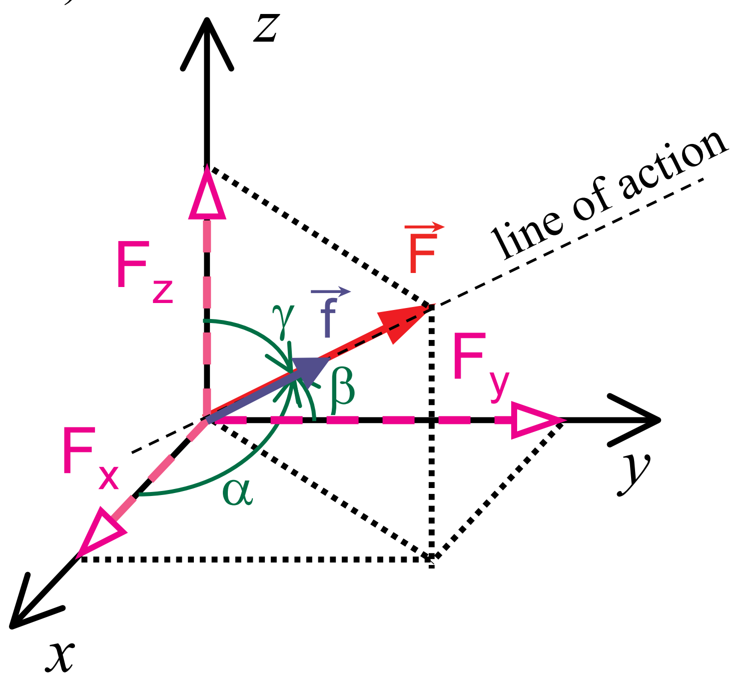
assuming constant mass:

$$m\frac{d\vec{v}}{dt} = m\vec{a} = \vec{F}$$

- basic unit: N (Newton)

$$1\text{N} = 1\ \text{kg m s}^{-2}$$

- force is a vector <u>tied</u> to its point of application (analysis of a force = analysis of a vector)



* components

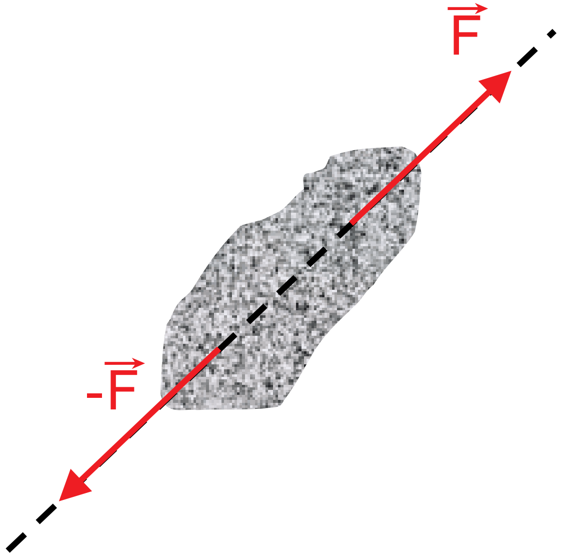
$$\vec{F} = \{ F_x; F_y; F_z \}$$

$$F_x = \vec{F} \cdot \vec{e}_1 = F \cos\alpha = F f_x$$
$$F_y = \vec{F} \cdot \vec{e}_2 = F \cos\beta = F f_y$$
$$F_z = \vec{F} \cdot \vec{e}_3 = F \cos\gamma = F f_z$$

* force magnitude:

$$F = (F_x^2 + F_y^2 + F_z^2)^{1/2}$$

3.1.3 Basic axioms



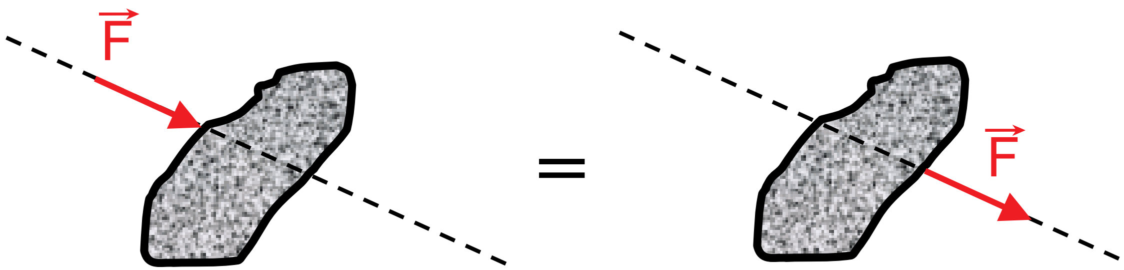
- arise from a vector character of a force

- <u>Axiom of equilibrium of forces</u>:

$$\vec{F} + (-\vec{F}) = \{ F_x+(-F_x); F_y+(-F_y); F_z+(-F_z) \}$$

$$= \{ 0; 0; 0 \} = \vec{0}$$

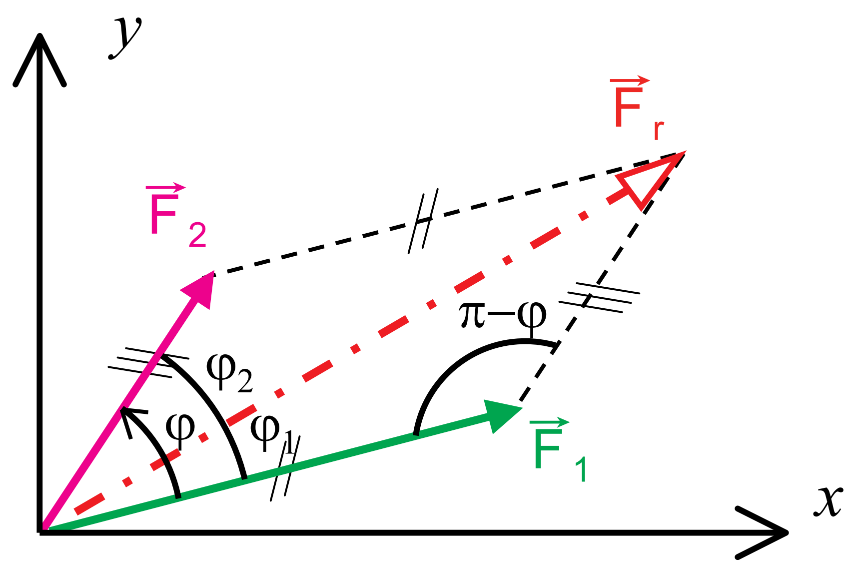
Translation of a point of application of a force along its line of action:
Effect of a force on a <u>rigid</u> body is not altered by shifting the force point of application along the force line of action



(rigid body ... force is a vector tied to its line of action)

- Axiom – parallelogram law:

resultant $\vec{F}_r$ of two forces $\vec{F}_1$ and $\vec{F}_2$



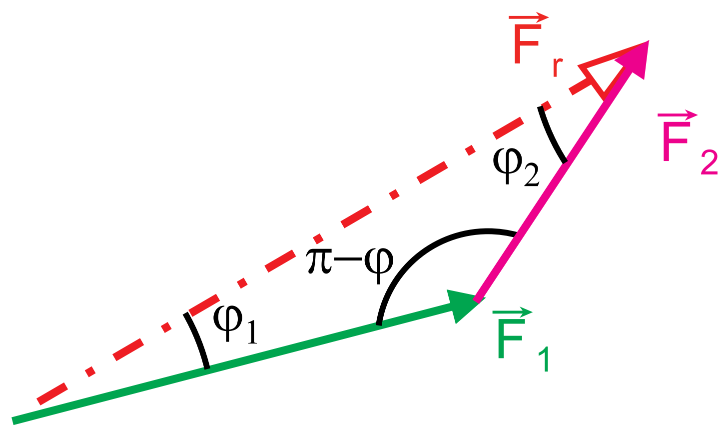
$$\vec{F}_r = \vec{F}_1 + \vec{F}_2$$
$$= \{ F_{1x}+F_{2x};\ F_{1y}+F_{2y}\}$$

From the law of cosines:

$$F_r = \sqrt{F_1^2 + F_2^2 - 2F_1 F_2 \cos(\pi - \varphi)}$$

$$\cos(\pi-\varphi) = -\cos\varphi$$

$$F_r = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos\varphi}$$



sine law:

$$\frac{\sin\varphi_1}{\sin(\pi-\varphi)} = \frac{F_2}{F_r} = \frac{\sin\varphi_1}{\sin\varphi}$$

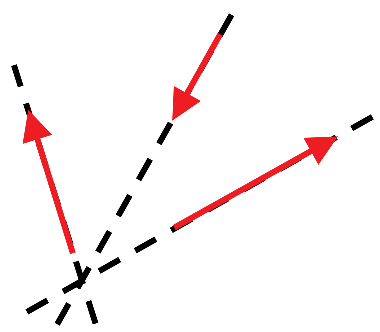
$$\frac{\sin\varphi_2}{\sin(\pi-\varphi)} = \frac{F_1}{F_r} = \frac{\sin\varphi_2}{\sin(\varphi)}$$

3.1.4 Concurrent system of forces

System of forces = set of forces acting on a body $\{\vec{F}_i\} = \{\vec{F}_1, \vec{F}_2, \vec{F}_3, \ldots, \vec{F}_n\}$

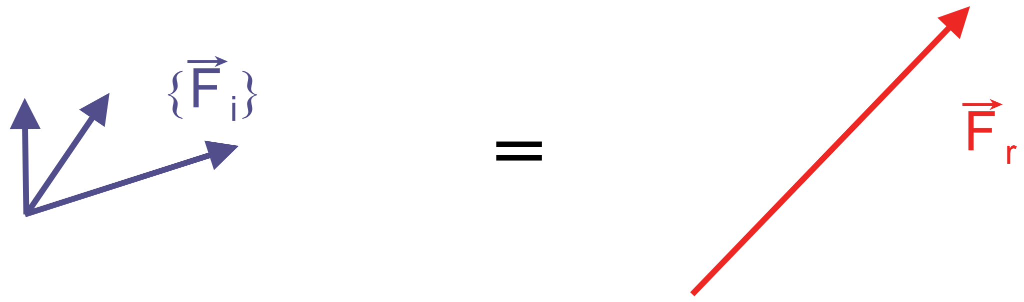
Concurrent system forces = system of forces where all lines of action intersect at one point

- space system
- plane system: all forces are found in a single plane

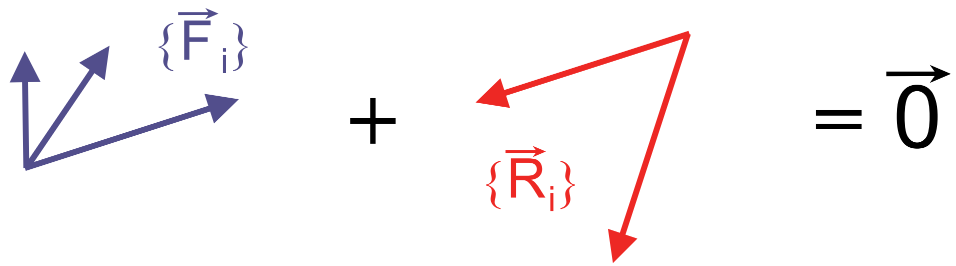


Task:

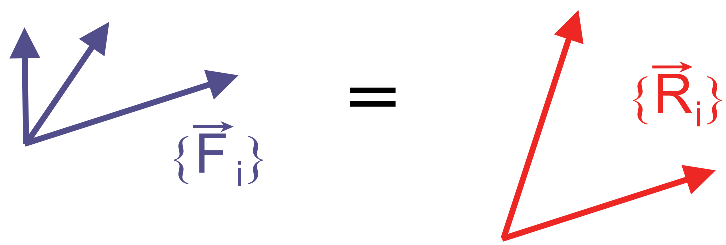
- The resultant of the concurrent system of forces: replacing the system of forces by a single force with the same effect



- equilibrium: canceling the effect of system $\{\vec{F}_i\}$ by adding a new system $\{\vec{R}_i\}$

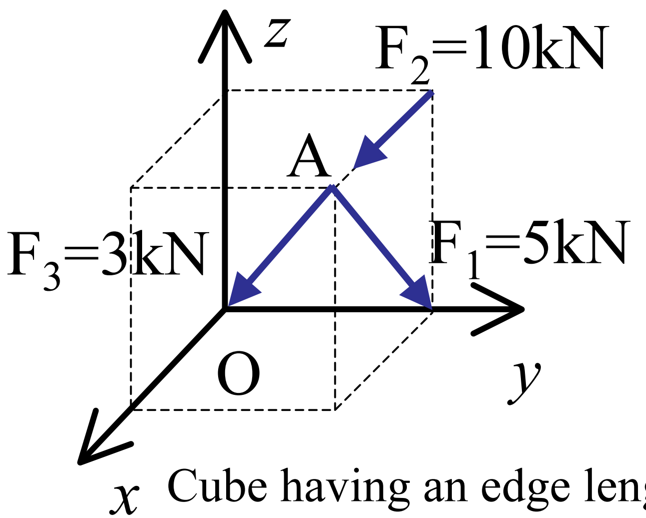


- equivalency: replacing the effect of system $\{\vec{F}_i\}$ by a different system $\{\vec{R}_i\}$



3.1.5 Concurrent system of forces in 3D

Ex.1: Determine the resultant of the concurrent system of forces



Cube having an edge length = 3m

1. Determine components

$F_{ix}=F_i \cos\alpha_i=F_i f_{ix}$

$F_{iy}=F_i \cos\beta_i=F_i f_{iy}$

$F_{iz}=F_i \cos\gamma_i=F_i f_{iz}$

$i=1,2,3$

2. Resultant

$\vec{F}_r=\vec{F}_1+\vec{F}_2+\vec{F}_3$

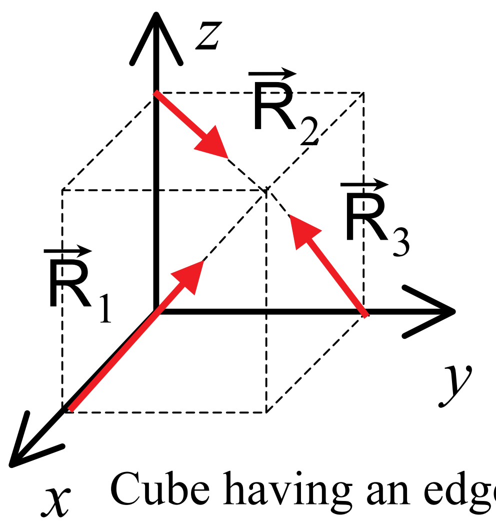
$F_{rx}=F_{1x}+F_{2x}+F_{3x}$

$F_{ry}=F_{1y}+F_{2y}+F_{3y}$

$F_{rz}=F_{1z}+F_{2z}+F_{3z}$

i	F	α	β	γ	fix=cosα	fiy=cosβ	fiz=cosγ	Fix	Fiy	Fiz
1	5	135	90	225	-0.707	0.000	-0.707	-3.536	0.000	-3.536
2	10	0	90	90	1.000	0.000	0.000	10.000	0.000	0.000
3	3				-0.577	-0.577	-0.577	-1.732	-1.732	-1.732
r								4.732	-1.732	-5.268

Ex.2: Bring the system of forces from Ex.1 into equilibrium by a system of 3 forces $\vec{R}_1$, $\vec{R}_2$, $\vec{R}_3$



Cube having an edge length = 3m

i	α	β	γ	fix=cosα	fiy=cosβ	fiz=cosγ
1	90	45	315	0.000	0.707	0.707
2	45	315	90	0.707	0.707	0.000
3	315	90	45	0.707	0.000	0.707

Equations of equilibrium

$$\vec{F}_r+\vec{R}_1+\vec{R}_2+\vec{R}_3=\vec{0}$$

$$x:\ F_{rx}+R_{1x}+R_{2x}+R_{3x}=0$$
$$y:\ F_{ry}+R_{1y}+R_{2y}+R_{3y}=0$$
$$z:\ F_{rz}+R_{1z}+R_{2z}+R_{3z}=0$$

$$x:\ F_{rx}+R_1\cos\alpha_1+R_2\cos\alpha_2+R_3\cos\alpha_3=0$$
$$y:\ F_{ry}+R_1\cos\beta_1+R_2\cos\beta_2+R_3\cos\beta_3=0$$
$$z:\ F_{rz}+R_1\cos\gamma_1+R_2\cos\gamma_2+R_3\cos\gamma_3=0$$

x: $4.732 + 0\,R_1 + 0.707\,R_2 + 0.707\,R_3 = 0$

y: $-1.732 + 0.707\,R_1 + 0.707\,R_2 + 0\,R_3 = 0$

z: $-5.268 + 0.707\,R_1 + 0\,R_2 + 0.707\,R_3 = 0$

$R_1 = 8.29703$ kN

$R_2 = -5.84724$ kN

$R_3 = -0.845827$ kN