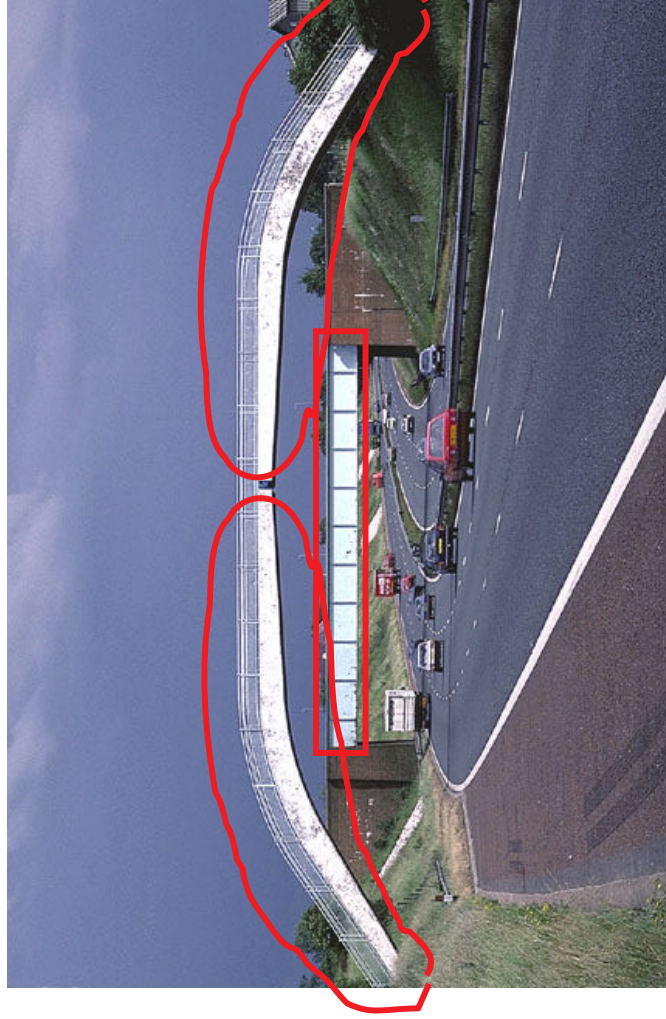


## 4. Statics of rigid bodies

### 4.1 Degree of freedom

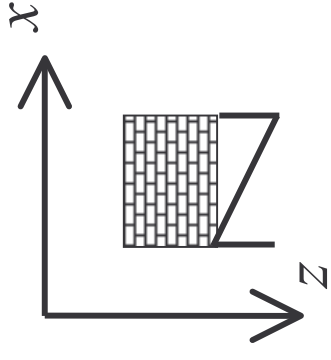
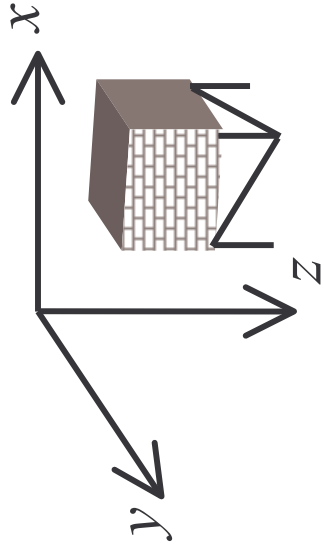
- The whole structure or its parts are idealized as rigid bodies



*foto: Godden Structural Engineering Slide Library  
Courtesy National Information Service for Earthquake Engineering, University of California, Berkeley*

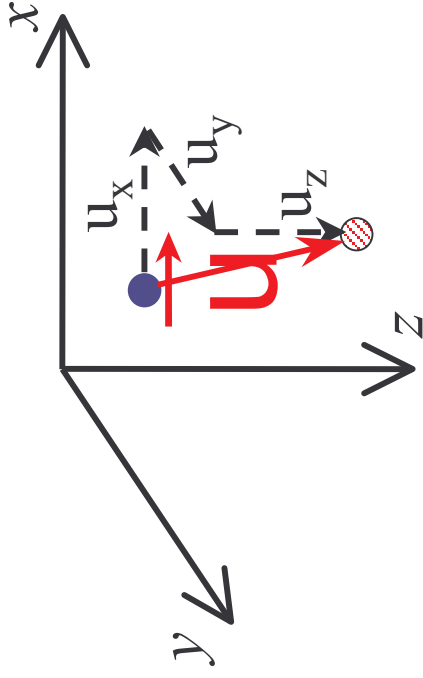
- Degrees of freedom ( $m$ ): serve to describe possible motion of rigid bodies<sub>1</sub>

*Note – the coordinate system is set such that the positive direction of the z-axis is aligned with the direction of gravity.*



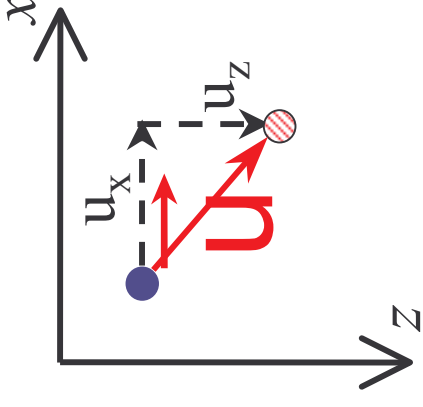
## Degrees of freedom of rigid particles

in space ( 3D)



$m = 3$  (displacements  $u_x, u_y, u_z$ )

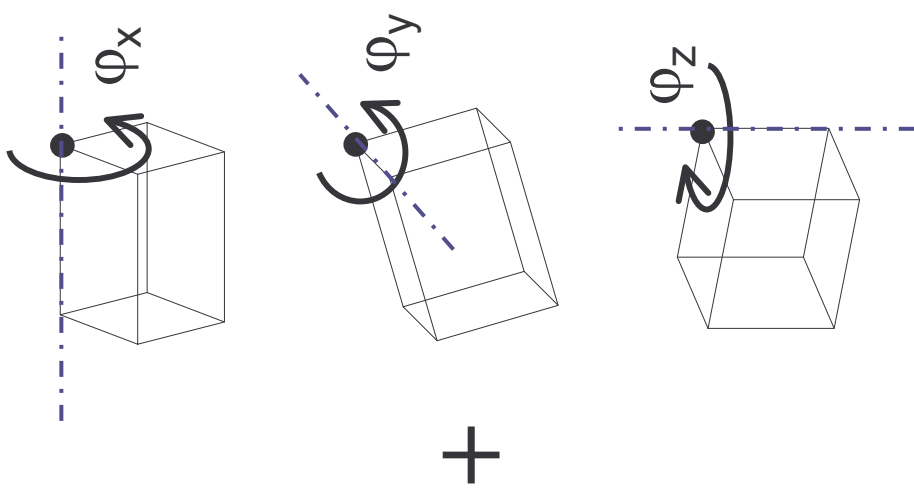
in plane (2D)



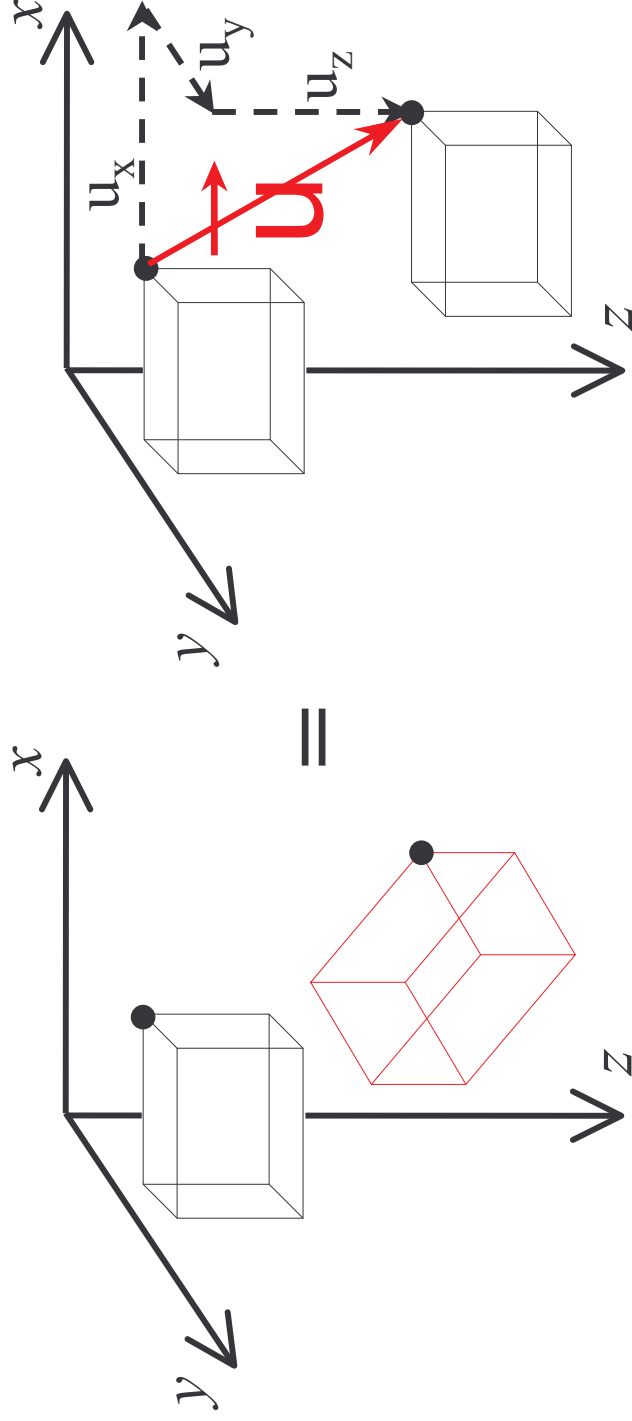
$m = 2$  (displacements  $u_x, u_z$ )

# Degrees of freedom of rigid bodies in 3D

rotations:

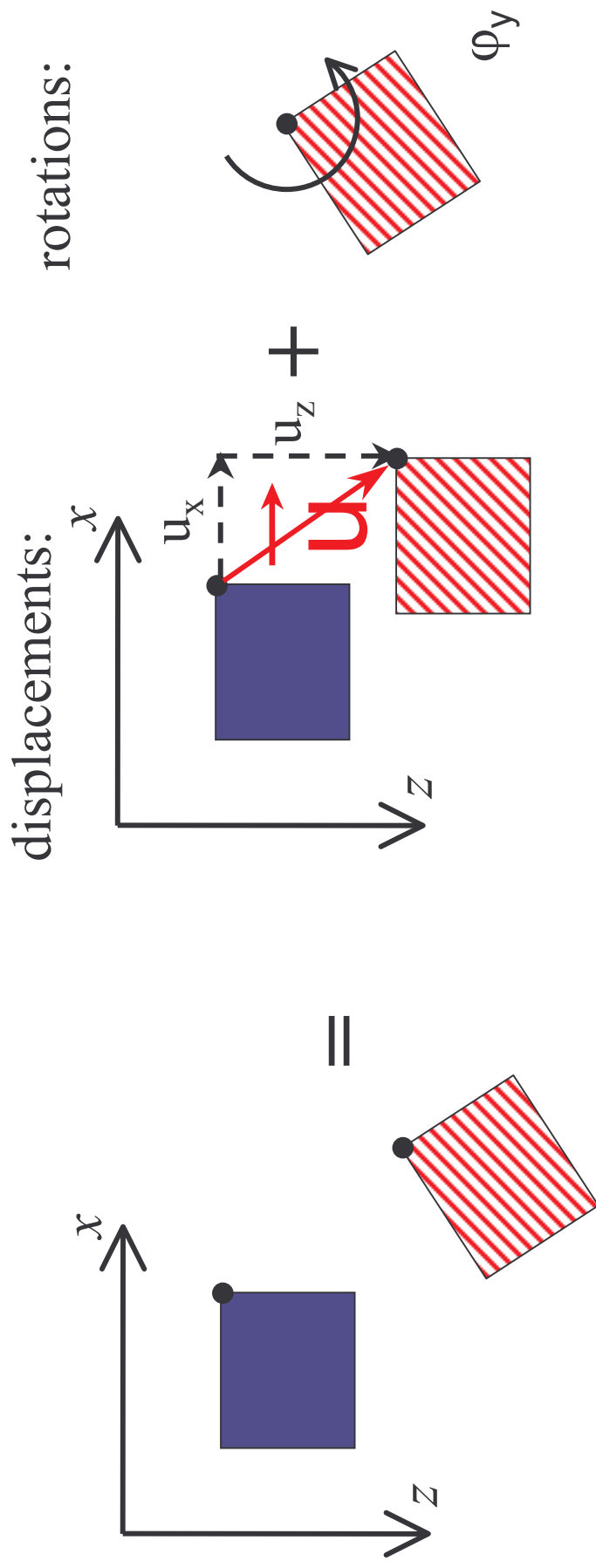


displacements:



$m=6$  (3 displacements  $u_x, u_y, u_z$  + 3 rotations  $\varphi_x, \varphi_y, \varphi_z$ )

## Degrees of freedom of rigid bodies in 2D



$$m = 3 \text{ (2 displacements } u_x, u_z + 1 \text{ rotation } \phi_y)$$

## 4.2 Supports, connections

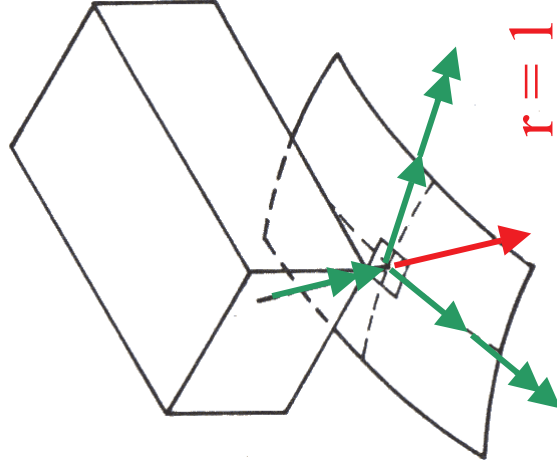
- devices that connect structural members or attach the structure to the foundation
- constrain motion of free objects, i.e., remove degrees of freedom (r)



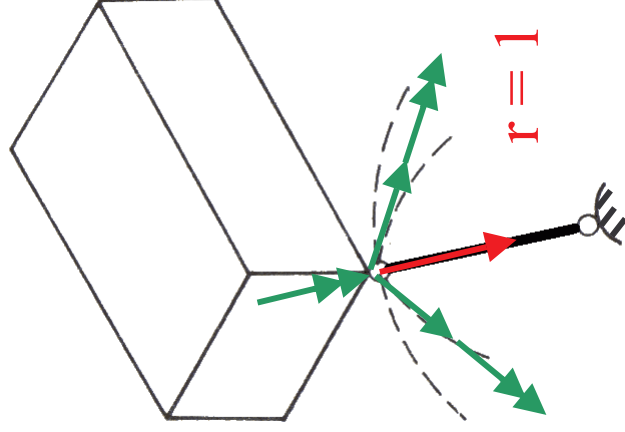
*foto: Godden Structural Engineering Slide Library  
Courtesy National Information Service for Earthquake Engineering, University of California, Berkeley*

## 4.2.1 Supports of rigid particles and rigid bodies in 3D

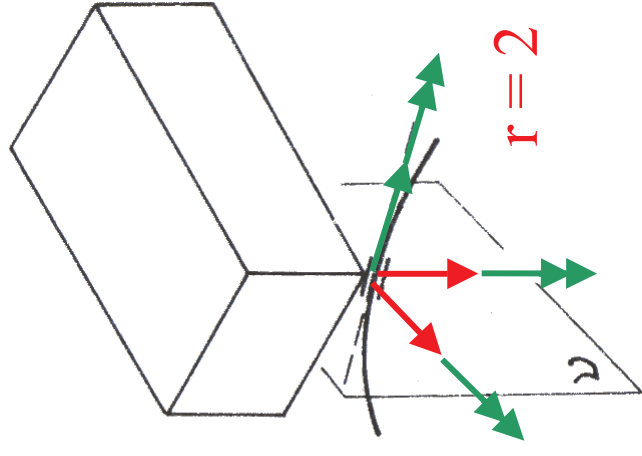
Smooth surface



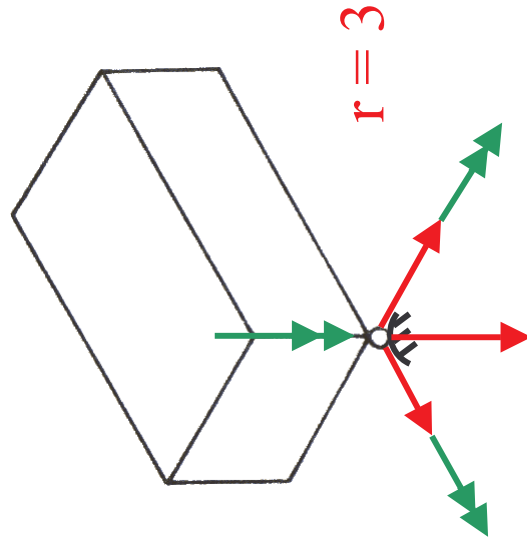
Rigid link



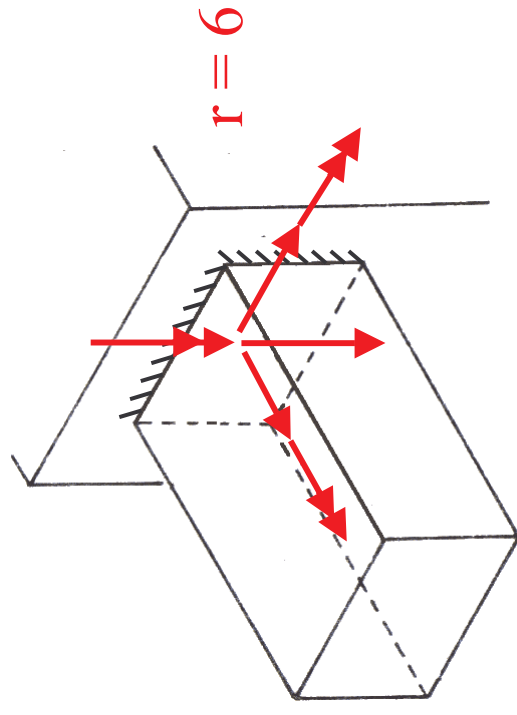
Ball bearing



Hinge – ball and socket

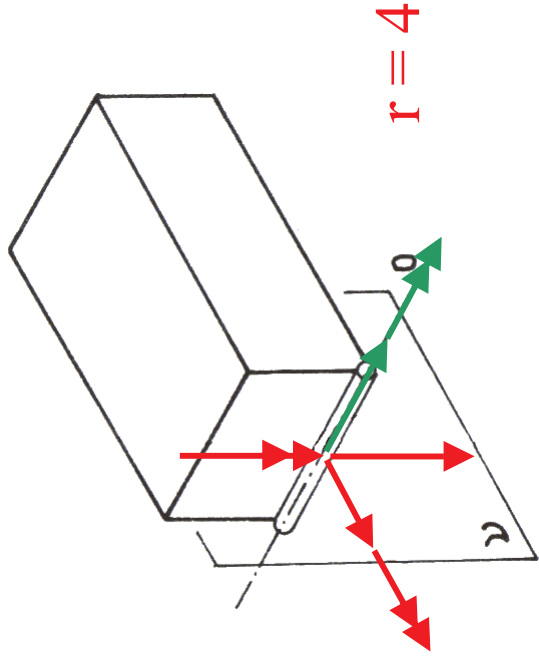


Fixed support  
(*not used for  $r$ .particle*)

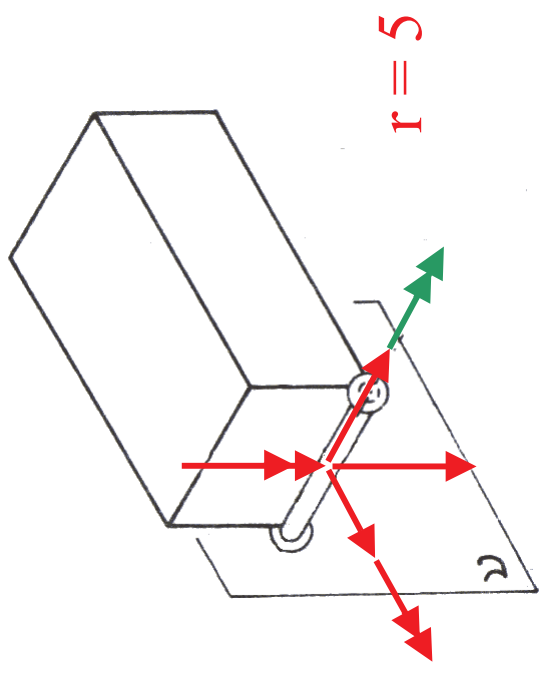




Journal bearing  
(*not used for  $r$ .particle*)

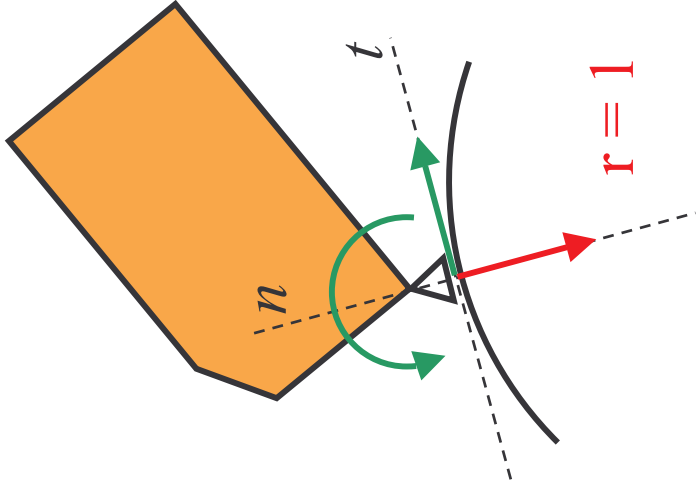


Thrust bearing  
(*not used for  $r$ .particle*)

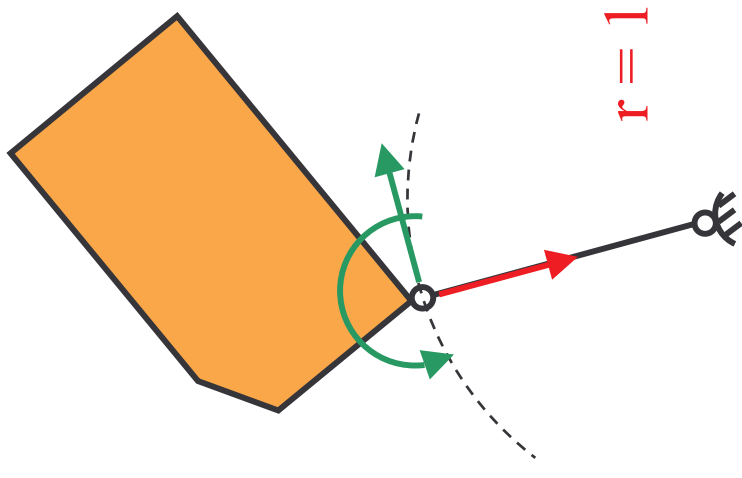


## 4.2.2.2 Supports of rigid particles and rigid bodies in 2D

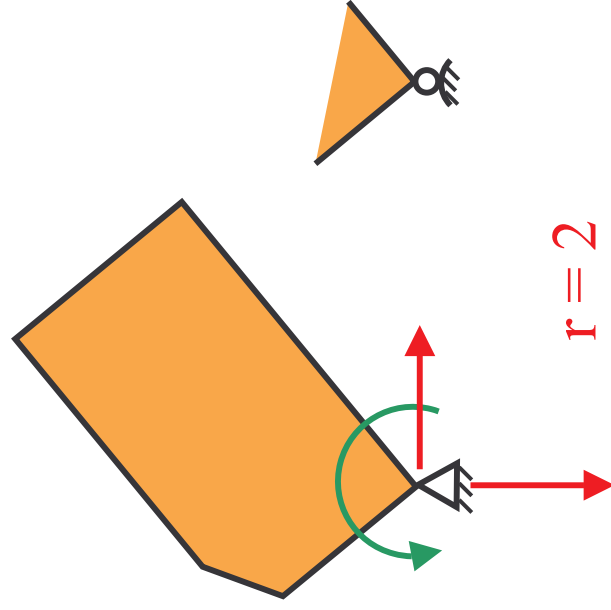
Smooth surface / Sliding pin



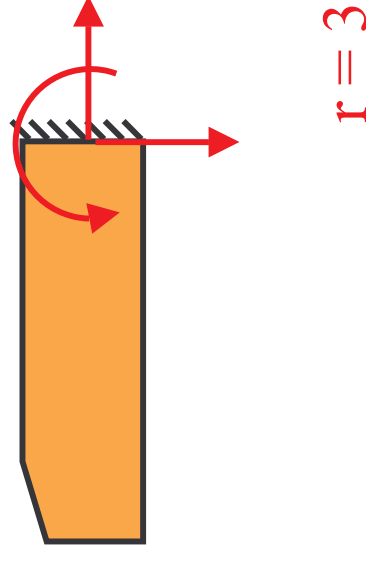
Rigid link



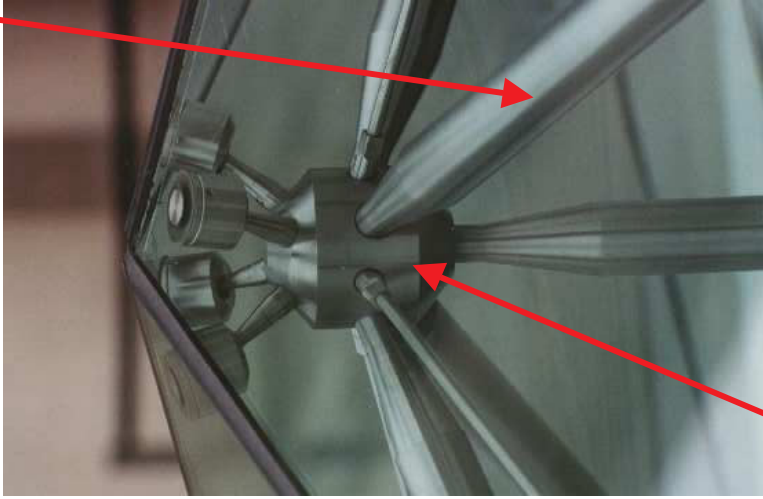
Hinge – smooth pin



Fixed  
(not used for *r.particle*)



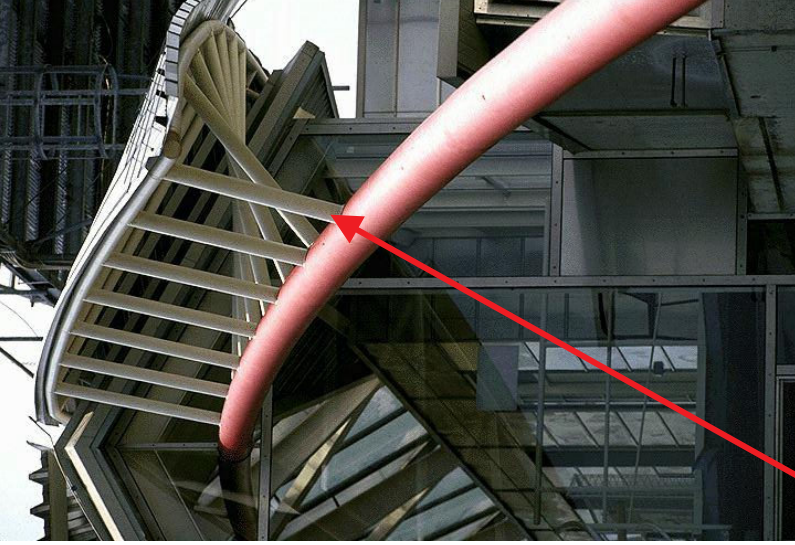
rigid link



rigid particle



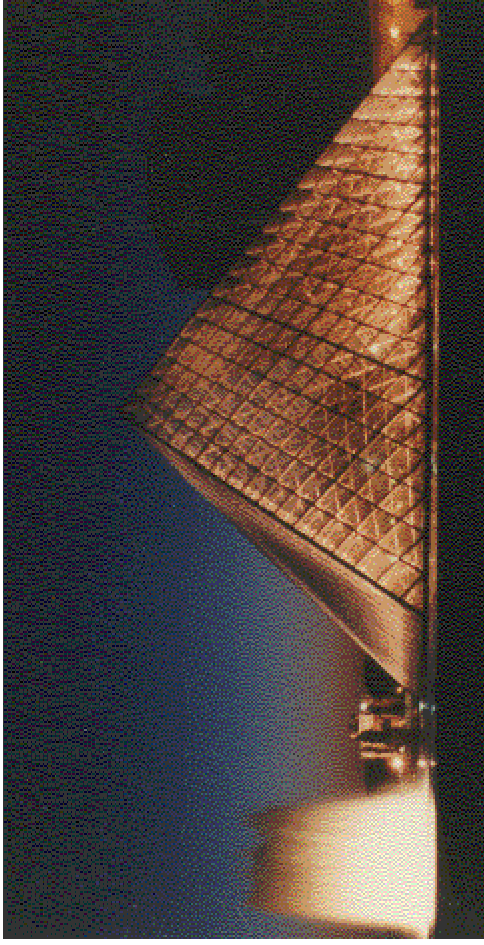
hinge



fixed



rigid link



source [www.triptyramid.com](http://www.triptyramid.com)

Structure composed of rigid  
links (trusses)

### 4.3 Statically determinate and indeterminate structures

| Degree of freedom                       | Support statically | Support kinematically | Note               |
|---|--------------------|-----------------------|--------------------|
| $m = r$<br>and<br>properly constrained  | determinate        | determinate           | Structure is fixed |
| $m < r$<br>and<br>properly constrained  | indeterminate      | overdeterminate       | Structure is fixed |
| $m > r$<br>or<br>improperly constrained | overdeterminate    | indeterminate         | Structure can move |

### Improperly supported structure:

- Although a number of constraints is sufficient to remove all degrees of freedom ( $m \leq r$ ), their arrangement does not prevent the structure from motion (either actual or infinitesimally small displacements and rotations)



- The reactions developed in supports (connections) that maintain equilibrium of a structure can not be determined.



- The system of equations of equilibrium is singular.

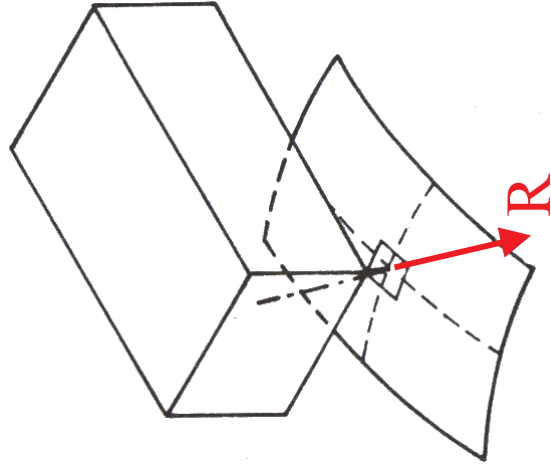
## 4.4 Effects of supports and connections

- forces exerted by the supports on the structure or its members are called **reactions** (either forces or static moments)
  - they act in the directions of removed degrees of freedom
- Sign convention
  - the positive reaction results in tensile force in the support (if the direction of the reaction is specified by the support)
  - positive reactions are aligned with the positive directions of the coordinate axis (the direction of the reaction is not specified)

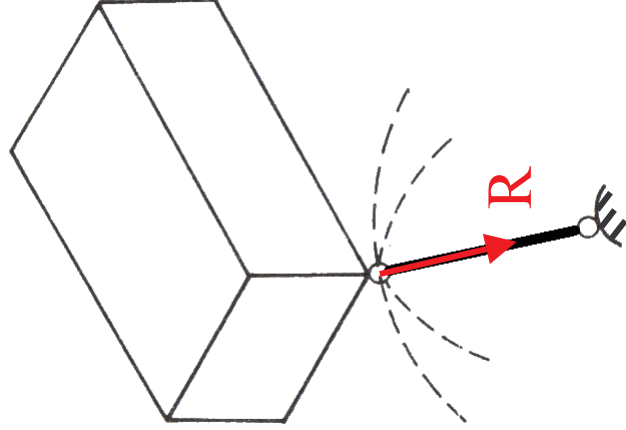


# 4.4.1 Reaction forces - rigid particles and rigid bodies in 3D

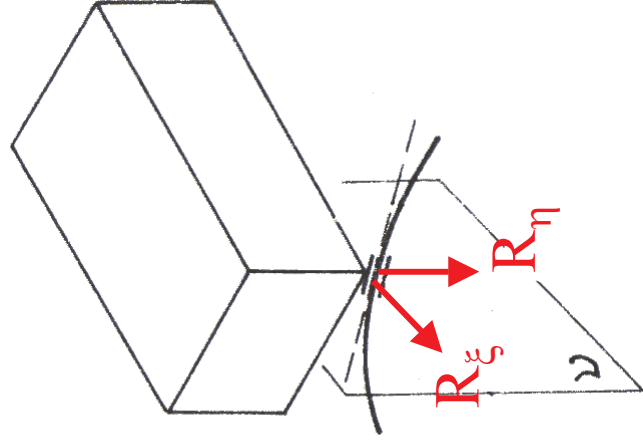
Smooth surface



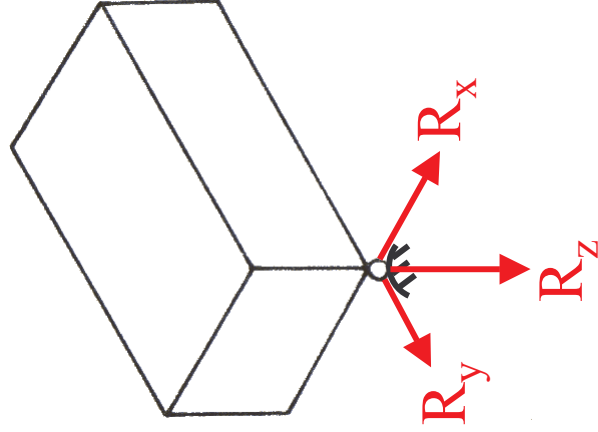
Rigid link



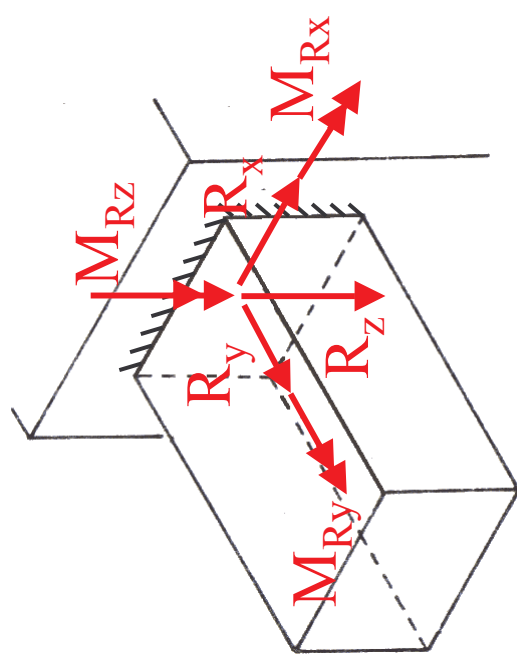
Ball bearing



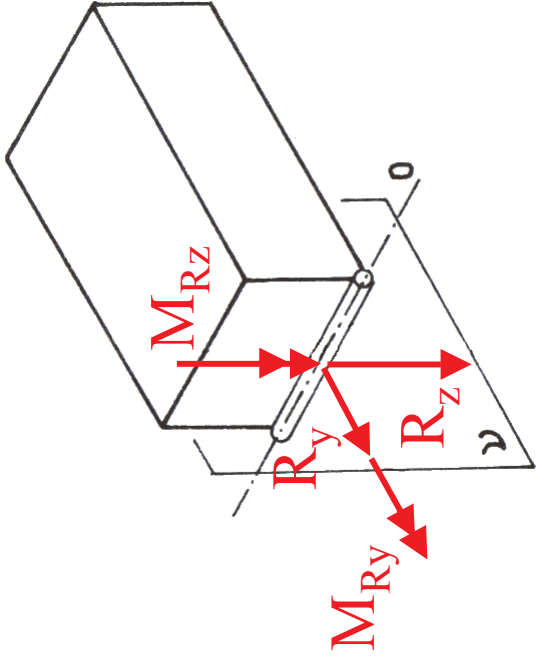
Hinge – ball and socket



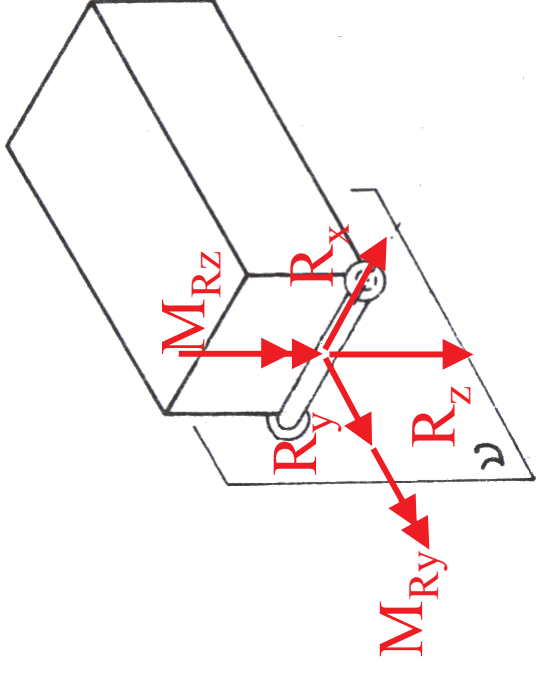
Fixed supports  
(*not used for r.particle*)



Journal bearing  
(*not used for r.particle*)

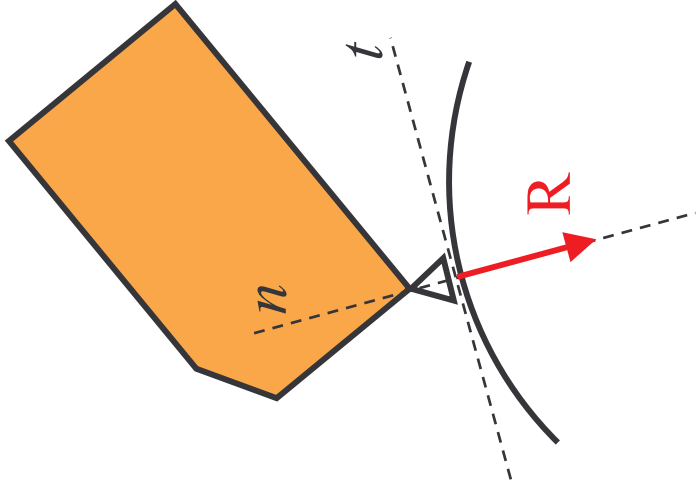


Thrust bearing  
(*not used for r.particle*)

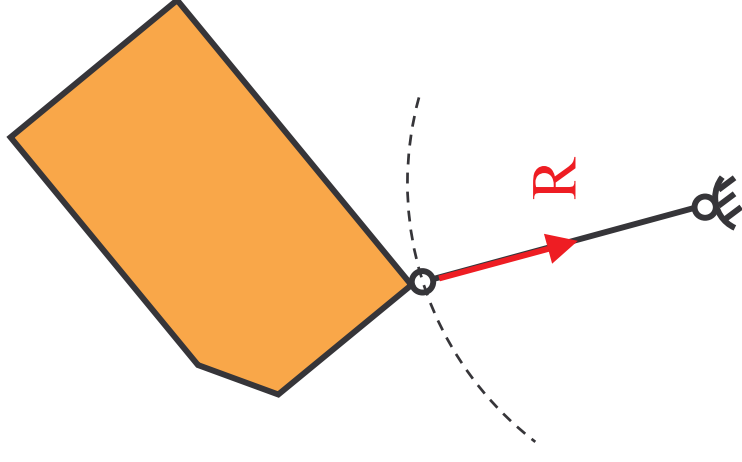


## 4.4.2 Reaction forces - rigid particles and rigid bodies in 2D

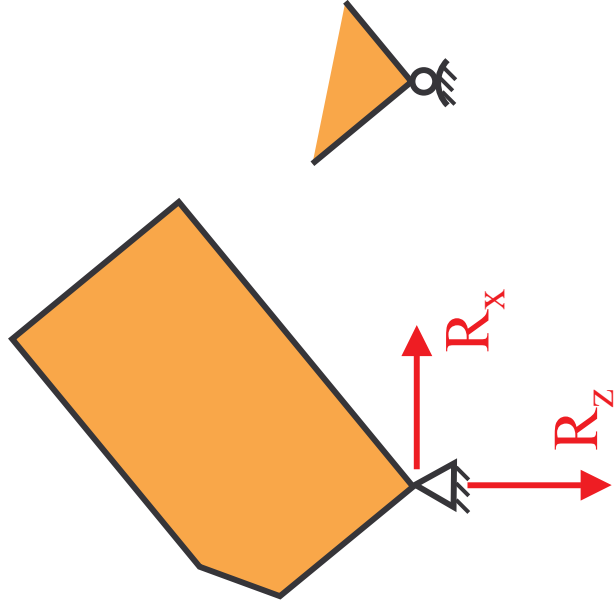
Smooth surface, sliding pin



Rigid link



Hinge – smooth pin



Fixed support  
(*not used for r.particle*)

