4.4 Equilibrium of statically determinate structures

Task:

- count the number of degrees of freedom and design the support system
- create a free body diagram (includes problem definition, model development and mathematical solution)

Assumptions - model development:

- the structure is idealized as a system of rigid particles and rigid bodies
- the structure is in the state of equilibrium

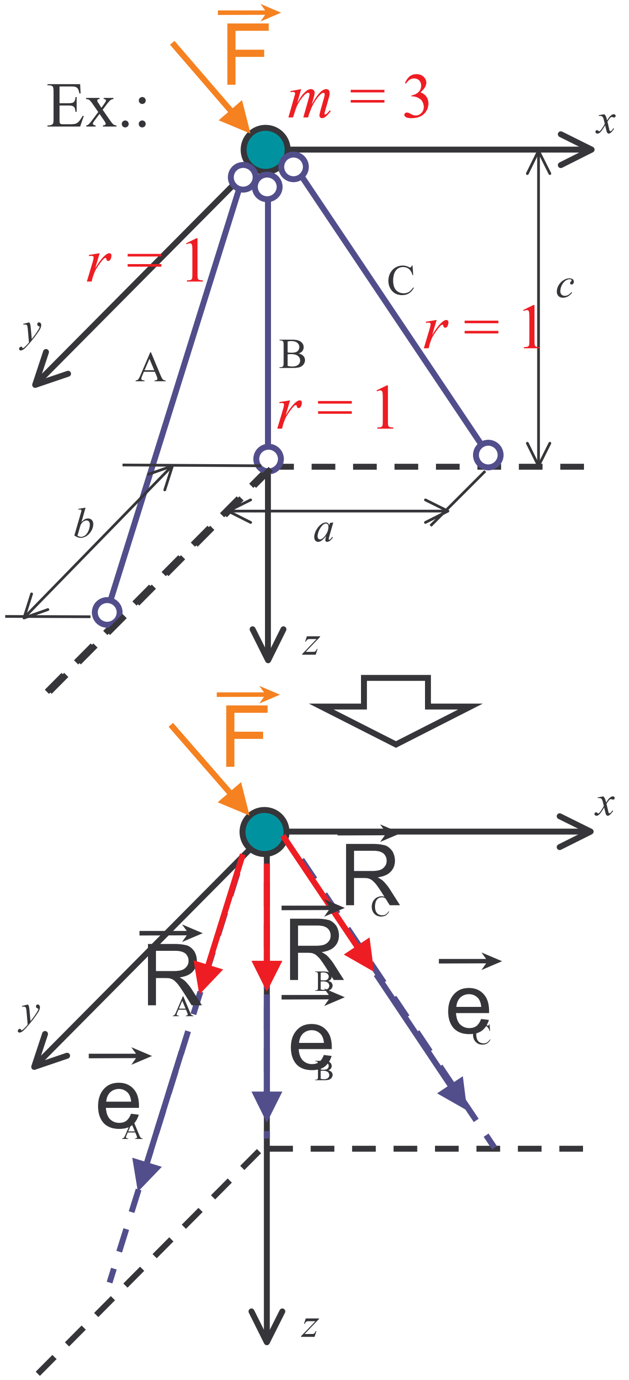
Solution:

- create a free body diagrams – separate individual structural members
- replace supports by corresponding reaction forces
- compute reaction forces such the whole structure as well as part are in equilibrium

4.4.1 Supports and reactions of rigid particles in 3D

- $m = r = 3$
- forces (applied loads and reactions) exerted on a rigid particle form a concurrent system of forces ($\Rightarrow$ 3 equations of equilibrium)

Ex.:



$$m = 3,\ r = 3\text{x}1,\ m = r$$

$$\vec{R}_A = \{R_A e_{Ax}, R_A e_{Ay}, R_A e_{Az}\}$$

$$\vec{R}_B = \{R_B e_{Bx}, R_B e_{By}, R_B e_{Bz}\}$$

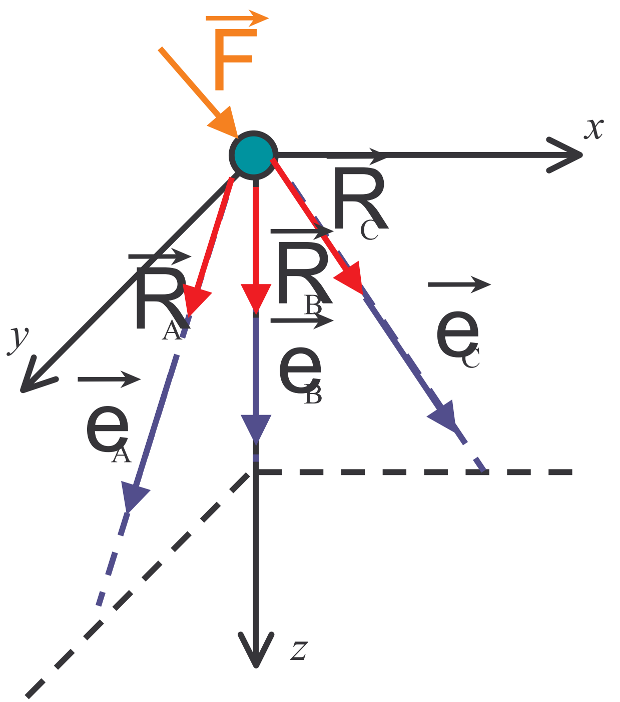
$$\vec{R}_C = \{R_C e_{Cx}, R_C e_{Cy}, R_C e_{Cz}\}$$

unit vectors:

$$\vec{e}_A = \left\{0, \frac{b}{\sqrt{b^2 + c^2}}, \frac{c}{\sqrt{b^2 + c^2}}\right\}$$

$$\vec{e}_B = \{0, 0, 1\}$$

$$\vec{e}_C = \left\{\frac{a}{\sqrt{a^2 + c^2}}, 0, \frac{c}{\sqrt{a^2 + c^2}}\right\}$$



Equations of equilibrium

$$\vec{F} + \sum_{i=A,B,C} R_i \vec{e}_i = 0$$

$$\text{x:}\quad F_x + R_A.0 + R_B.0 + R_C.\frac{a}{\sqrt{a^2+c^2}} = 0$$

$$\text{y:}\quad F_y + R_A.\frac{b}{\sqrt{b^2+c^2}} + R_B.0 + R_C.0 = 0$$

$$\text{z:}\quad F_z + R_A.\frac{c}{\sqrt{b^2+c^2}} + R_B.1 + R_C.\frac{c}{\sqrt{a^2+c^2}} = 0$$

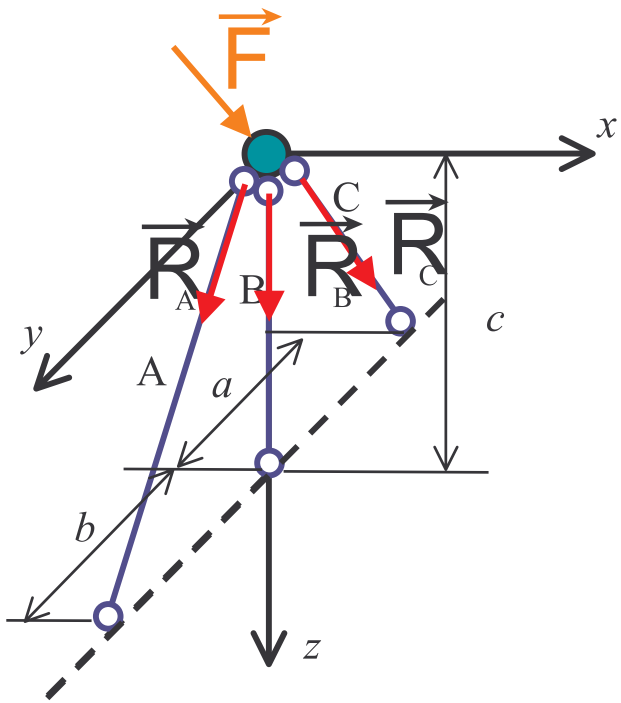
Solve for 3 unknowns R_A, R_B a R_C

H.W.: Solve for a = 1 m, b = 2 m, c = 4 m, $\vec{F} = \{10, 5, 20\}$ (kN)

- improperly supported structure:

➢ e.g. lines of action of all reaction forces are found in the same plane

Ex.:



Equations of equilibrium

$$x: F_x + R_A.0 + R_B.0 + R_C.0 = 0$$

$$y: F_y + R_A.\frac{b}{\sqrt{b^2+c^2}} + R_B.0 - R_C.\frac{a}{\sqrt{a^2+c^2}} = 0$$

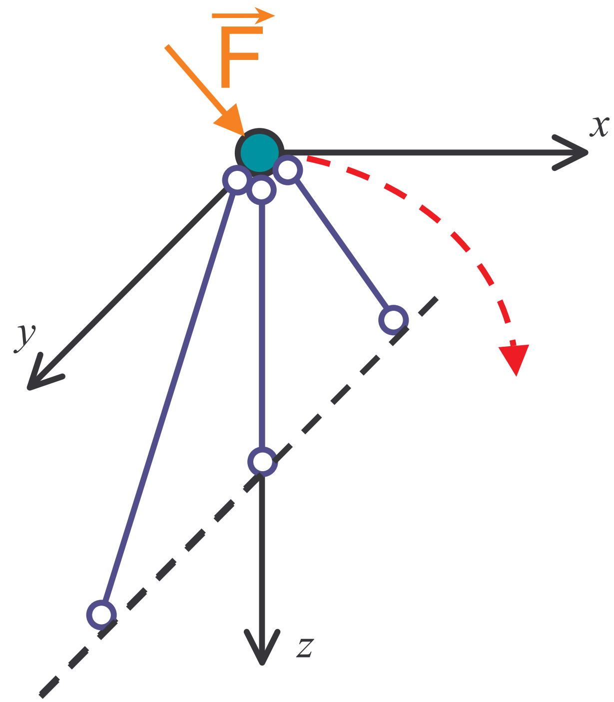
$$z: F_z + R_A.\frac{c}{\sqrt{b^2+c^2}} + R_B.1 + R_C.\frac{c}{\sqrt{a^2+c^2}} = 0$$

Determinant:

$$\begin{vmatrix} 0 & 0 & 0 \\ \frac{b}{\sqrt{b^2+c^2}} & 0 & -\frac{a}{\sqrt{a^2+c^2}} \\ \frac{c}{\sqrt{b^2+c^2}} & 1 & \frac{c}{\sqrt{a^2+c^2}} \end{vmatrix} = 0$$

Solution does not exists (the system is singular)

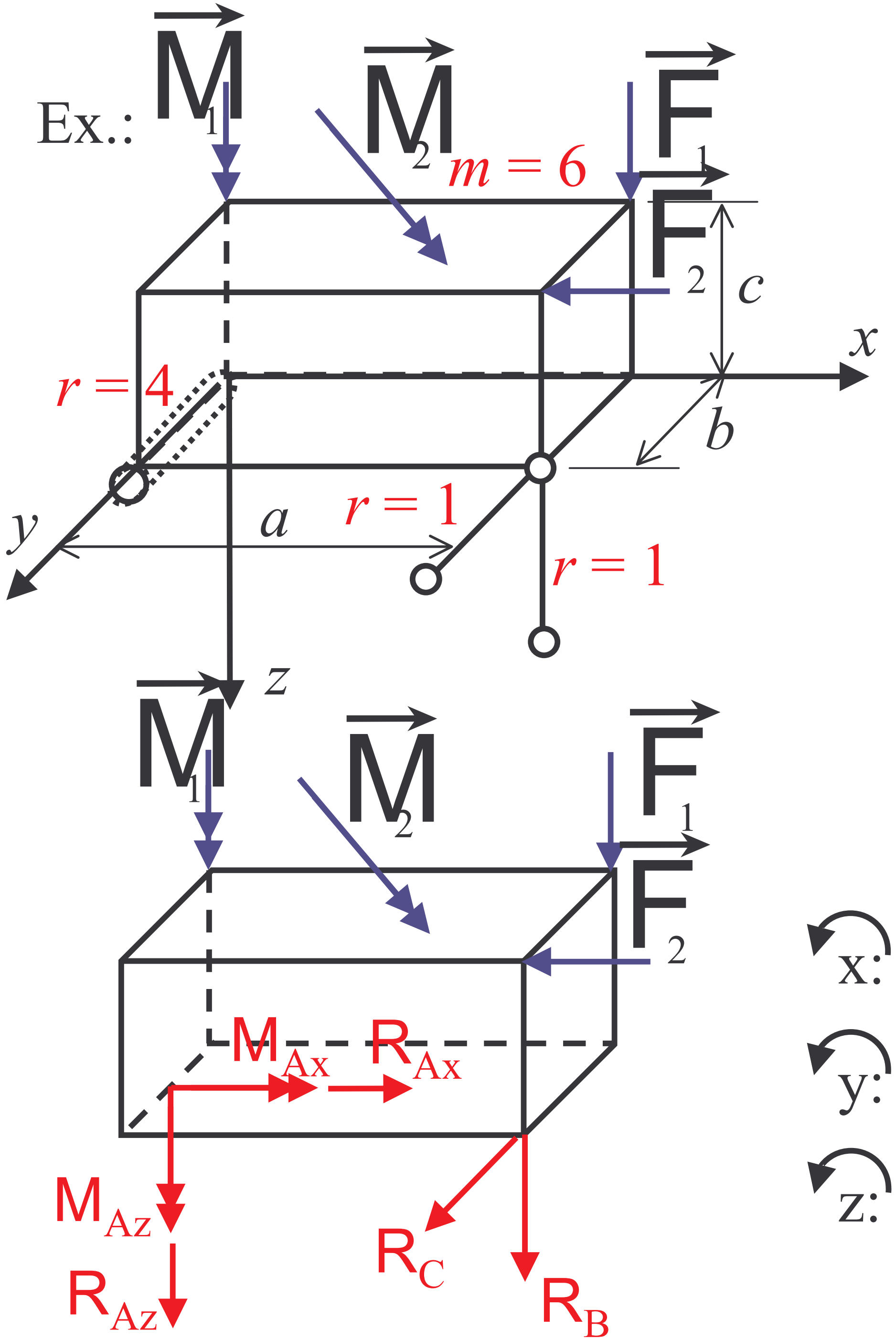
- interpretation



4.4.2 Supports and reactions of rigid bodies in 3D

- $m = r = 6$
- Forces (applied loads and reactions) exerted on a body form a general system of forces ($\Rightarrow$ 6 equations of equilibrium)

Ex.:



$m = 6$, $r = 1\text{x}4 + 2\text{x}1$, $m = r$

Equations of equilibrium

$$\rightarrow x:\quad \underbrace{\sum F_{ix}}_{\text{(load)}} + \underbrace{\sum R_{jx}}_{\text{(reactions)}} = 0$$

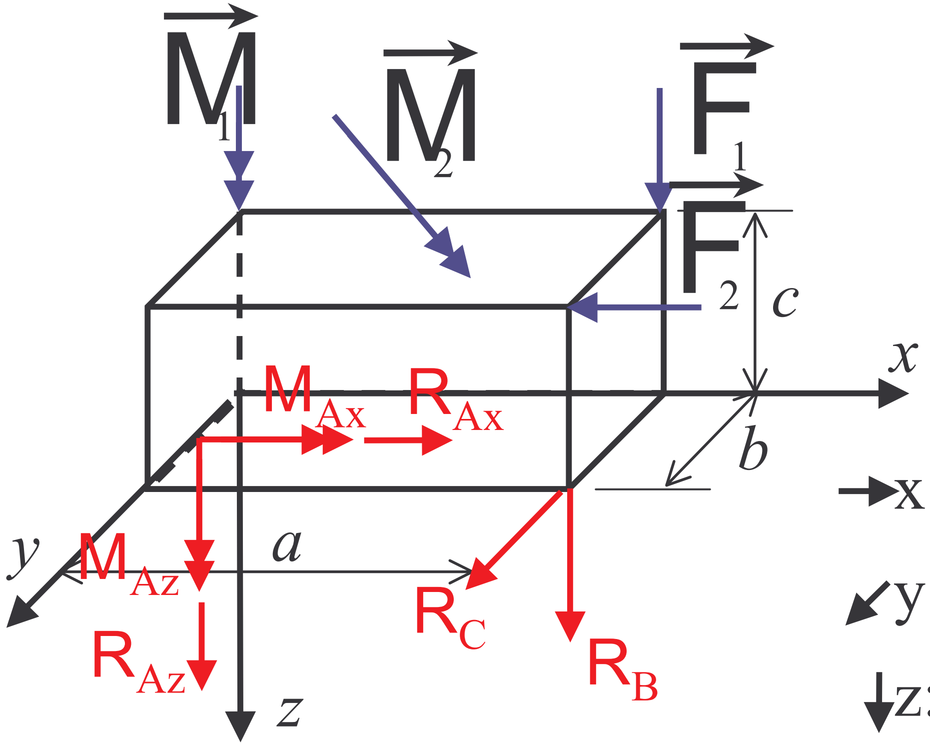
$$\swarrow y:\quad \sum F_{iy} + \sum R_{jy} = 0$$

$$\downarrow z:\quad \sum F_{iz} + \sum R_{jz} = 0$$

$$x:\quad \sum M^F_{ix} + \sum M_{kx} + \sum M^R_{jx} + M^R_{Ax} = 0$$

$$y:\quad \sum M^F_{iy} + \sum M_{ky} + \sum M^R_{jy} + M^R_{Ay} = 0$$

$$z:\quad \underbrace{\sum M^F_{iz} + \sum M_{kz}}_{\text{(load)}} + \underbrace{\sum M^R_{jz} + M^R_{Az}}_{\text{(reactions)}} = 0$$



Equations of equilibrium

$\rightarrow$ x: $$-F_2 + R_{Ax} = 0$$

$\swarrow$ y: $$0 + R_C = 0$$

$\downarrow$ z: $$F_1 + R_{Az} + R_B = 0$$

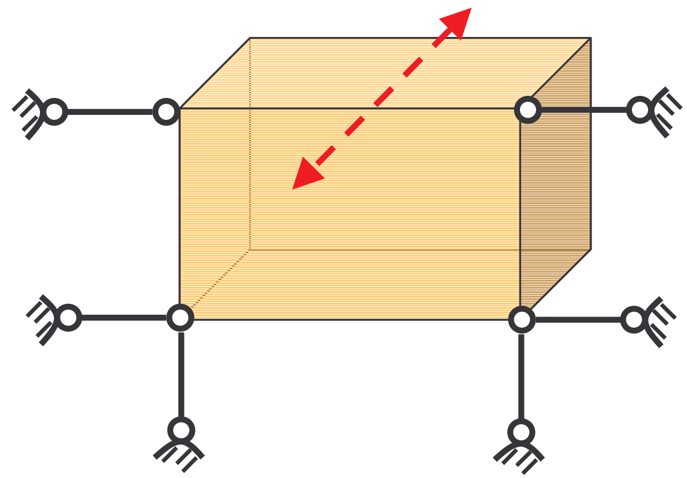
x: $$M_{2x} + R_{Az}.\frac{b}{2} + R_B.b + M_{Ax} = 0$$

y: $$M_{2y} - F_1.a + F_2.c - R_B.a = 0$$

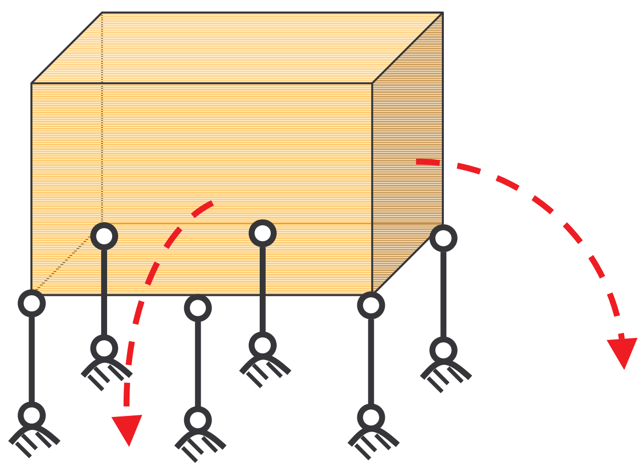
z: $$M_1 + M_{2z} + F_2.b - R_{Ax}.\frac{b}{2} + R_c.a + M_{Az} = 0$$

H.W.: Solve for a = 6 m, b = 2 m, c = 0.2 m,
F_1 = 15 kN, F_2 = 5 kN, M_1 = 6 kNm, $\vec{M}_2 = \{10, 5, 20\}$ (kNm)

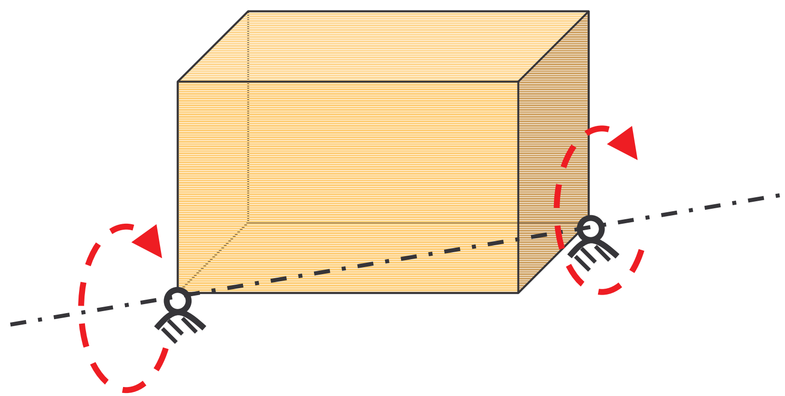
- improperly supported structure, e.g.:
  - ➢ lines of action of all reactions are found in the same plane

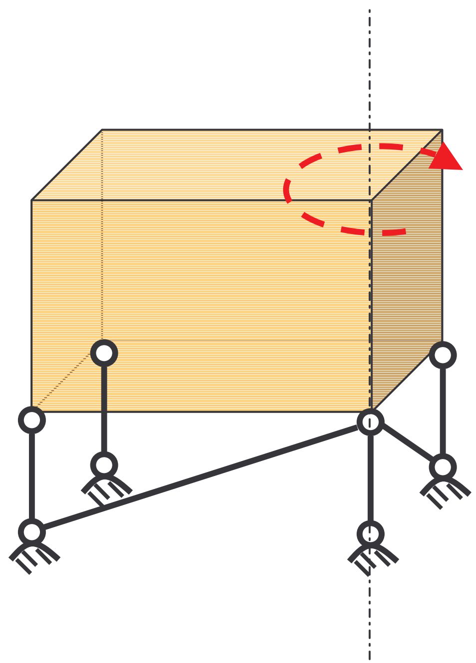


  - ➢ lines of actions of all reactions are parallel



- improperly supported structure, cont.:
  - ➢ lines of action of all reactions cross so called zero line (reactions give zero moment with respect to this line)



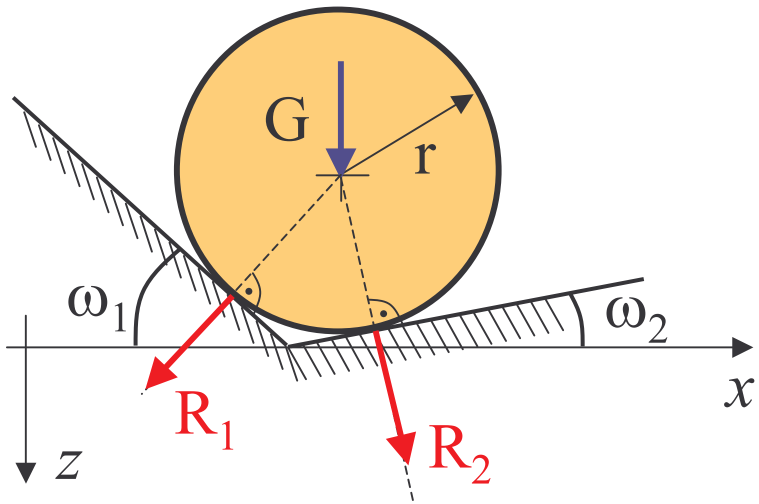


H.W.: Using rigid links support a 3D box such that the structure is properly constrained

4.4.3 Supports and reactions of a rigid particle in 2D

- $m = r = 2$
- forces (applied loads and reactions) exerted on a rigid particle form a concurrent system of forces ($\Rightarrow$ 2 equations of equilibrium)

Př: Determine forces exerted by two smooth surfaces on a circular disk. The disk weight is G.



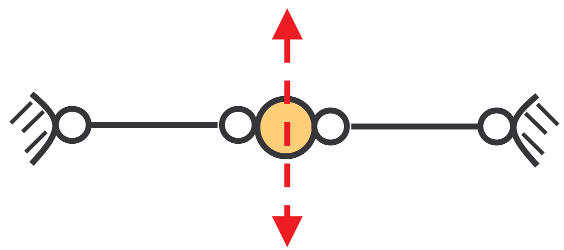
Equations of equilibrium:

$$x: \quad -R_1 \sin\omega_1 + R_2 \sin\omega_2 = 0$$

$$z: \quad G + R_1 \cos\omega_1 + R_2 \cos\omega_2 = 0$$

H.W.: Solve for $\omega_1 = 45^\circ$, $\omega_2 = 30^\circ$ a $G = 500$ N.

- improperly supported structure:
  - e.g., lines of action of all forces are found in the same line



4.4.4 Supports and reactions of rigid bodies in 2D

- $m = r = 3$
- Forces (applied loads and reactions) exerted on a body form a general system of forces ($\Rightarrow$ 3 equations of equilibrium)

3 <u>independent</u> equations of equilibrium:

a) 2 force equations (x, z) and 1 moment equation w.r.t. the origin

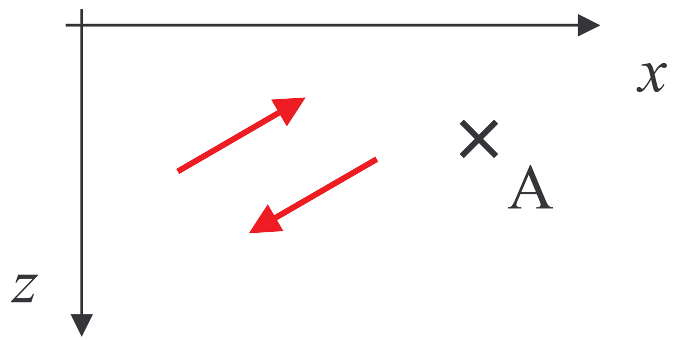
$$\sum F_x = 0 \qquad \sum F_z = 0 \qquad \sum M_0 = 0$$

... reduction from 3-D

b) 2 force eqns. (x, z) and 1 moment equation w.r.t. an arbitrary point A

$$\underbrace{\sum F_x = 0 \qquad \sum F_z = 0}_{\text{couple}} \qquad \sum M_A = 0$$

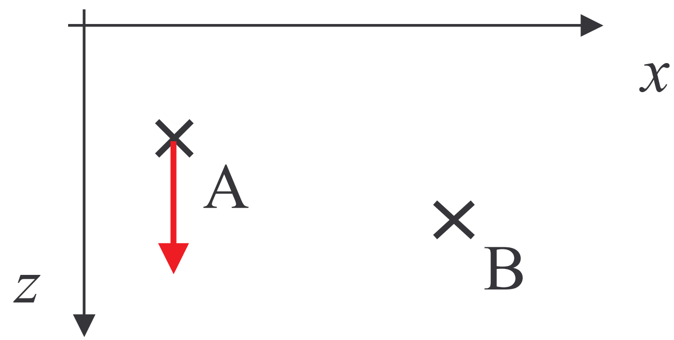
Moment w.r.t. an arbitrary point = $\vec{0}$



c) 1 force eq. (x) and 2 moment eqns. w.r.t. to two points A, B $(\overline{AB} \nparallel z)$

$$\underbrace{\sum F_x = 0 \qquad \sum M_A = 0}_{\text{force in A, parallel with } z} \qquad \sum M_B = 0$$

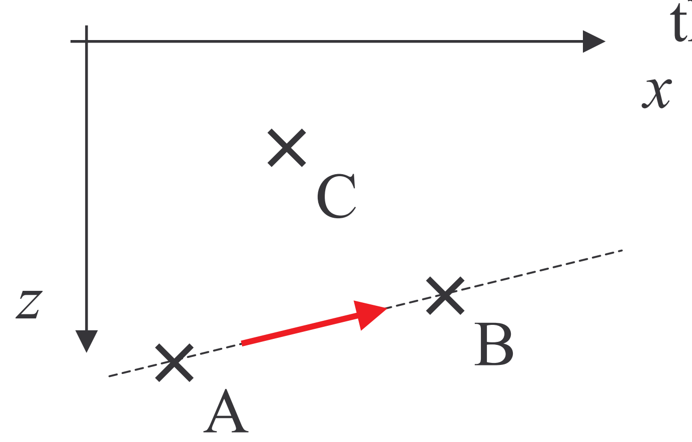
moment w.r.t. an arbitrary point B found off the line of action of force through point A= $\vec{0}$



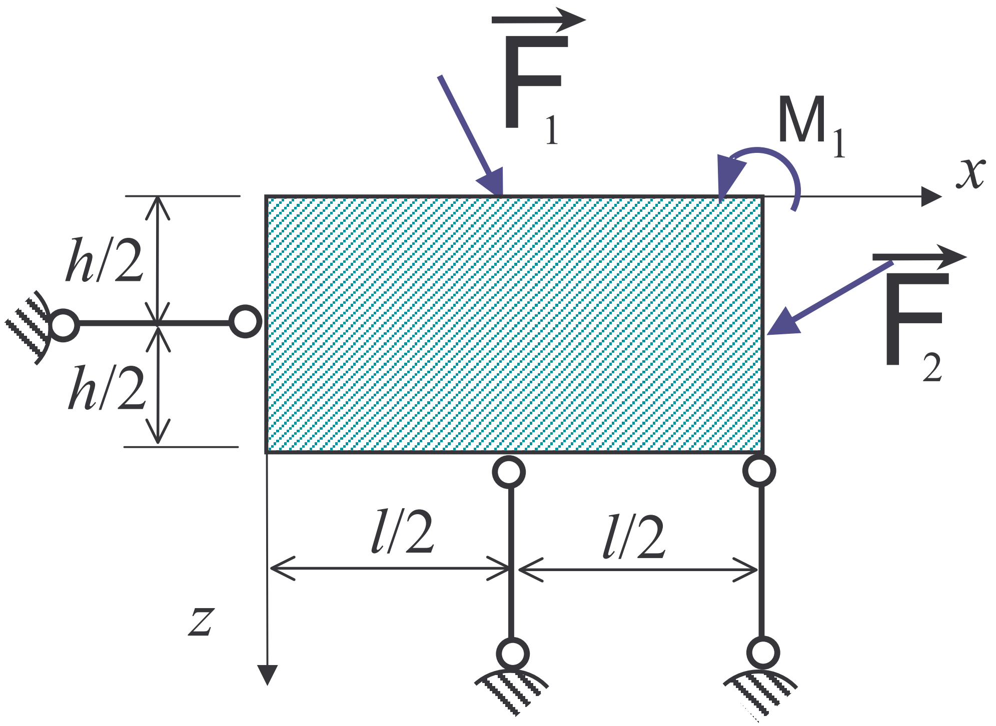
d) 3 moment eqns. w.r.t. arbitrary 3 points A, B, C not found along the same line

$$\underbrace{\sum M_A = 0 \qquad \sum M_B = 0}_{\text{force along line AB}} \qquad \sum M_C = 0$$

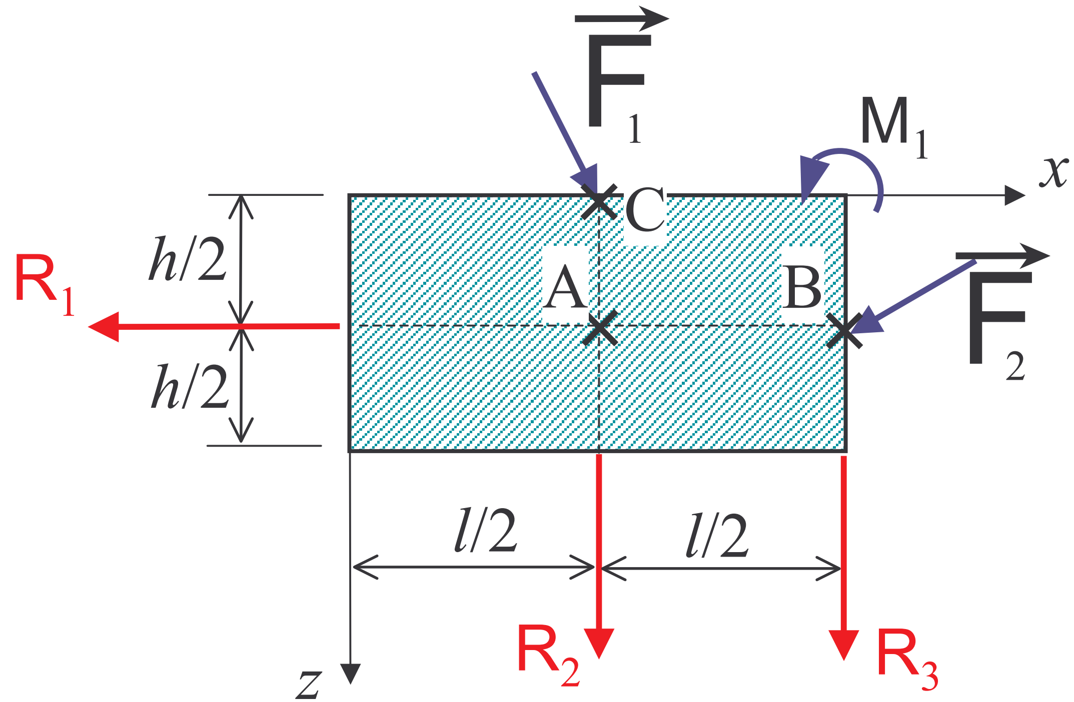
moment w.r.t. an arbitrary point C found off the line of action of this force = $\vec{0}$



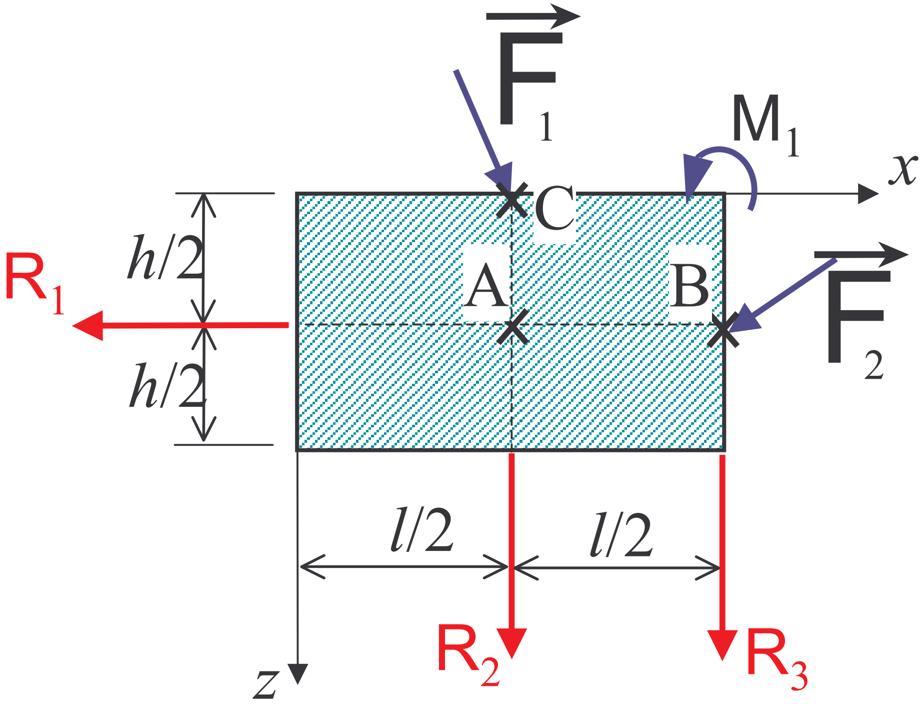
Use such equations of equilibrium so that the resulting system of equations is as simple as possible !



F1
M1
x
F2
h/2
h/2
l/2
l/2
z



F1
M1
x
C
A
B
F2
R1
h/2
h/2
l/2
l/2
z
R2
R3

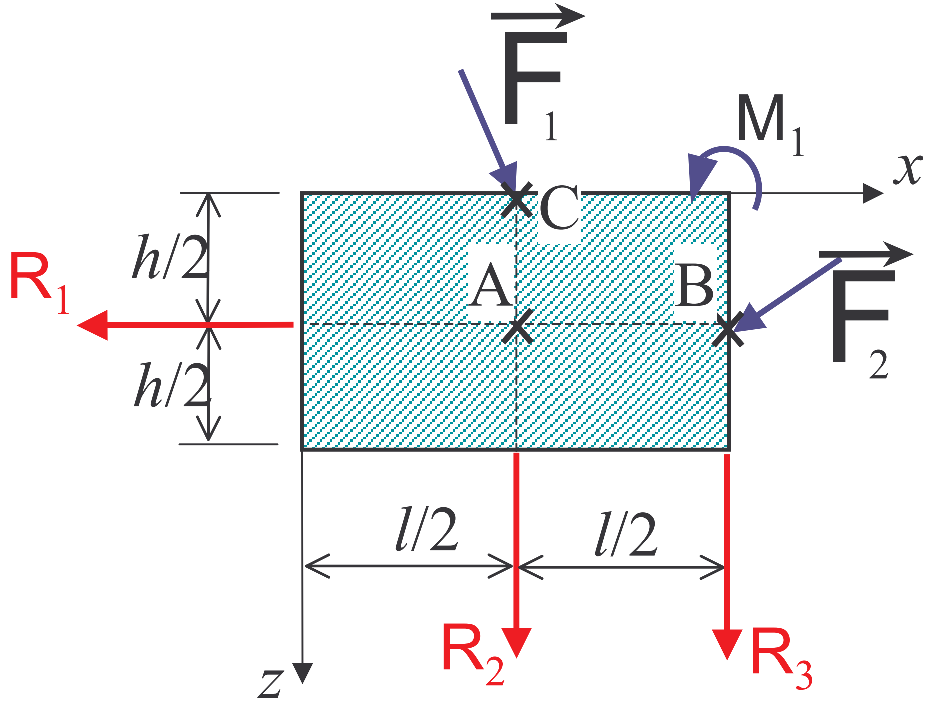


Equations of equilibrium (a):

$$\rightarrow x: \quad F_{1x} + F_{2x} - R_1 = 0$$

$$\downarrow z: \quad F_{1z} + F_{2z} + R_2 + R_3 = 0$$

$$\widehat{0}: \quad M_1 + M_0^{F_1} + M_0^{F_2} - R_1 \frac{h}{2} - R_2 \frac{l}{2} - R_3 l = 0$$

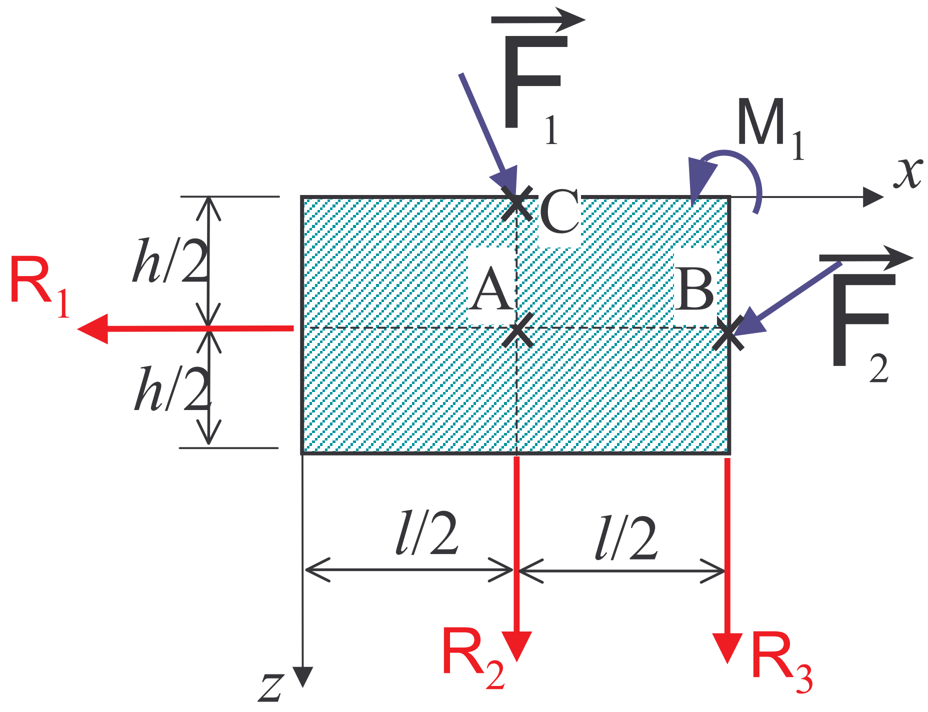


Equations of equilibrium (b):

$$\rightarrow x: \quad F_{1x} + F_{2x} - R_1 = 0$$

$$\downarrow z: \quad F_{1z} + F_{2z} + R_2 + R_3 = 0$$

$$\widehat{A}: \quad M_1 + M_A^{F_1} + M_A^{F_2} - R_3 \frac{l}{2} = 0$$

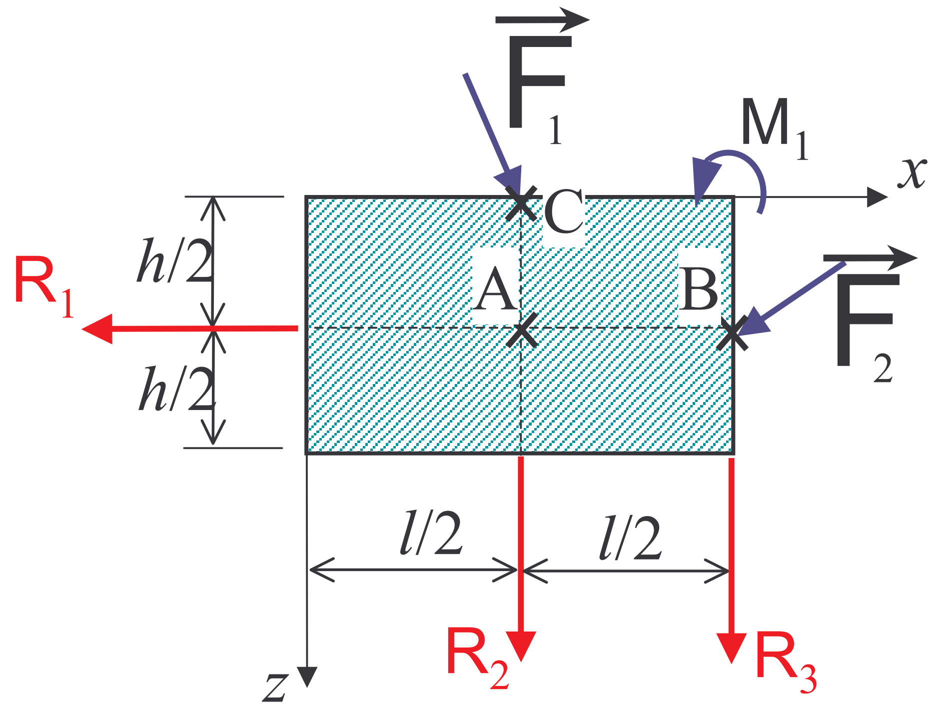


Equations of equilibrium (c):

$$\rightarrow x: \quad F_{1x} + F_{2x} - R_1 = 0$$

$$\widehat{A}: \quad M_1 + M_A^{F_1} + M_A^{F_2} - R_3 \frac{l}{2} = 0$$

$$\widehat{B}: \quad M_1 + M_B^{F_1} + \cancel{M_B^{F_2}} + R_2 \frac{l}{2} = 0$$



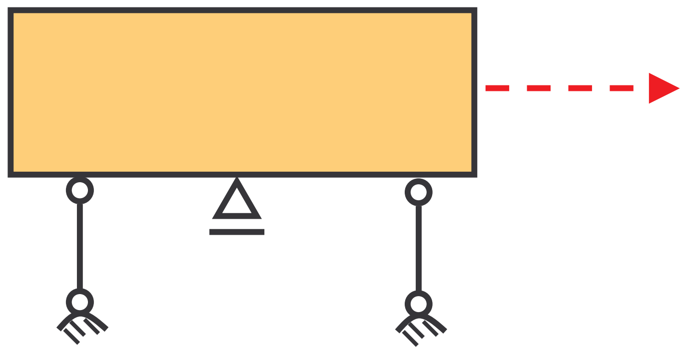
Equations of equilibrium (d):

$$\widehat{A}: \quad M_1 + M_A^{F_1} + M_A^{F_2} - R_3 \frac{l}{2} = 0$$

$$\widehat{B}: \quad M_1 + M_B^{F_1} + \cancel{M_B^{F_2}} + R_2 \frac{l}{2} = 0$$

$$\widehat{C}: \quad M_1 + \cancel{M_C^{F_1}} + M_C^{F_2} - R_1 \frac{h}{2} - R_3 \frac{l}{2} = 0$$

- improperly supported structure, e.g.:
  - lines of action of all reactions are parallel to each other



  - lines of action of all forces intersect at one point